Purchase Obligations and Hedging*

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Abstract

Commodity price shocks have negative consequences for developed economies that rely heavily on imported materials. Consequently, firms employ risk-management instruments to reduce their exposure. In this paper we study how the use of supply contracts by firms can shape the transmission of commodity price shocks to aggregate variables. We focus on purchase obligations, which are supply contracts with fixed prices for the delivery of goods in future periods. We rely on a novel dataset to document two empirical findings. First, we find a large exposure reduction to commodity price risk for firms using these contracts; our estimates suggest a reduction of about 27% compared with non-users. Second, sector output and labor compensation have a smaller negative correlation with commodity prices when firms trade larger contracts. We assess the aggregate quantitative role of these contracts by introducing and calibrating a tractable general equilibrium model. We measure the contribution of purchase obligations to dampening the aggregate transmission of commodity price shocks by constructing a counterfactual in which firms are not allowed to trade these contracts. Our results show that when firms engage in purchase obligations, real consumption has a relative response of 25% less to a 10% commodity price shock.

Keywords: commodity price shocks, hedging, purchase obligations
JEL Codes: E32, F44, G32

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1 Introduction

A long-standing research question in international economics concerns the aggregate consequences of commodity price shocks in countries that rely heavily on imported materials. In general, these price shocks are associated with economic hardship characterized by a reduction in employment and consumption (e.g., **?**). Due to lack of available data, this literature has overlooked the fact that the firms that demand these inputs use financial instruments to hedge against the volatility of their prices.[1] These risk-management policies could reduce the negative aggregate consequences of commodity price shocks.

In this paper we study how firms' risk management policies can have implications for the transmission of commodity price shocks to aggregate variables. We use a novel dataset on purchase obligations for U.S. public manufacturing firms to study the relationship between firm dynamics and the propagation of commodity price shocks. Purchase obligations are supply contracts for future purchases of goods that include fixed prices and quantities. Firms rely on these contracts to reduce their exposure to commodity price risk. In general equilibrium, the use of these contracts can provide insurance to the whole economy by reducing the negative impact of these shocks.

We use the dataset to discuss three empirical findings. Starting from firms' risk-management strategies, we show that these contracts allow them to reduce their exposure to commodity price risk. Following the empirical literature on hedging, we define exposure as the elasticity of firm value with respect to commodity prices.[2] We measure commodity prices by constructing a material input price index by sector (NAICS-3) using the Economic Census of 2012 and commodity prices from the BLS and World Bank. We use the terms "input prices" and "commodity prices" interchangeably, because these input price indexes only contains commodities.

We show empirically that firms that use purchase obligations have a lower exposure to these material input price shocks. Our estimates suggest that companies that use purchase obligations have a relative lower exposure of 27% compared with non-users. The second empirical finding suggests that the use of these contracts can dampen the transmission of commodity price shocks to aggregate variables. We show that the correlation of input prices and sector aggregate variables, such as labor income and output, is lower when the sector aggregate value of purchase obligations is large.

[1] Recent finance papers only focus on individual firms or small sectors (e.g., **?**). For a review of commodity hedging in finance, see **?**.

[2] See **?**, for example.

We then introduce a tractable theoretical model that showcases these findings. It extends a basic trade model, similar to **?**, with no entry and exit and featuring purchase obligations. There will be two countries (home and rest of the world) in which home firms require labor, capital and imported material inputs to produce goods. The home economy takes as given the prices for imported inputs. Input materials can be bought on the spot market or one period ahead using a purchase obligation contract. Both markets are assumed to have a perfectly elastic supply.

The model has two stages. In the first stage, the spot price is unknown and companies choose their purchase obligations conditional on the distribution of spot prices and their expected profit in the second stage. These purchase obligations consist of a future contract for the delivery of a fixed quantity of input materials at a fixed price on the next stage.

In the second stage, each firm produces a differentiated variety using labor and imported material inputs. They sell their output to a representative firm that bundles all varieties and sells to domestic or foreign consumers. The key difference with standard trade models is that firms borrow funds to cover their costs, facing financial constraints. We model these along the lines of **?**. Firms use profits as collateral for borrowing. If profits are low, firms are required to pay additional expenses over borrowed funds (distress costs) that we model as an interest-rate premium. This is particularly damaging for firms when input prices are high.

Purchase obligations are modeled as an input supply forward contract that yields positive income if spot prices are larger than future prices. These financial assets can increase income precisely when firms are more constrained and compensate for the decline in profits from a surge in input prices. By increasing the quantity of assets traded, the firm can raise income and reduce expected interest payments on externally borrowed funds.

In general equilibrium, the use of purchase obligations can reduce the transmission of input price shocks to aggregate variables. An increase in input prices reduces sales, and firms face larger distress costs. They raise prices and reduce labor demand to offset the burden of external financing. This implies that wages and real consumption react more to input price shocks. When firms ex ante buy purchase obligations, they can reduce their external financing; therefore, this can dampen the transmission to aggregate variables.

The model matches empirical facts of our dataset across the firms' size distribution. In particular, there is a size threshold for the use of purchase obligations. Small firms do not hedge due to fixed costs. Only large firms benefit from hedging, because they can reduce their expected distress costs by buying purchase obligations and justify the fixed costs. Along the intensive margin, the model predicts constant hedging ratio. As the constraints are defined relative to profits over costs, all hedging firms choose the same hedging ratio.

We use the model to study quantitatively aggregate implications. We simulate the model to match stylized facts of the U.S. economy and compare the solution to a counterfactual in which firms are not allowed to trade purchase obligations. We find that there are positive aggregate effects on real consumption and employment. We measure welfare gains by computing the equivalent variation between these two models. On one hand, for low spot prices, there are welfare losses due to negative income from the future contract. On the other hand, for large spot prices, firms can increase income from the forward operation and reduce their distress costs. Our results show that the representative consumer is willing to sacrifice 0.024% of her consumption to allow firms to trade these supply contracts.

Finally, we compute the relative response of aggregate variables to input price shocks between the calibrated model with purchase obligations and the counterfactual. We find a strong reduction of aggregate transmission when firms trade purchase obligations. For example, when firms engage in purchase obligations, real consumption has a relative response of 25% less to a 10% commodity price shock.

Related Literature. This paper is connected to three strands of the literature. First, the international economics literature has studied, in great detail, the transmission of commodity price shocks, mainly focusing on oil price shocks. Examples are **?**; **?**; **?**; **?**; **?**; **?** and **?**. Our paper is mostly related to **?**, who study oil shocks in several developed economies. Their empirical results show a large decrease in output and employment after an oil shock. They also find a substantial increase in nominal wages and consumer prices. On the other hand, **?** estimate a Structural Vector Auto Regression (SVAR) model to understand the contribution of commodity shocks to domestic business cycles. Their results show that commodity prices explain about a third of the variance of output and 20% of the variances of consumption and investment. In this paper, we allow for the possibility that the transmission of commodity price shocks could be partially offset through the use of risk-management tools. In particular, we study the role of supply contracts with fixed prices as a hedging device. Our results suggest that this new channel plays an important part in dampening the propagation of these shocks.

Second, risk-management has been extensively studied in finance. The Modigliani-Miller theorem suggests that hedging will not increase firm value, and therefore firms should avoid using these instruments. However, the literature has shown several deviations from this theorem. These include risk-averse managers, convex tax schedules, and external financing costs.[3] This paper will follow **?**, in the sense that firms will face external financing costs. In our model, the use of purchase obligations could reduce their external financing burden and increase profits. Another set of related papers study the relationship between financing and risk management, but only focus on individual firms or an industry equilibrium. See for example **?**, **?**, **?**, **?**, **?**, **?**. Our paper will extend their analysis by studying aggregate implications of risk-management policies.

Empirical studies in the finance literature have focused primarily on producers of commodities, such as oil, gas, and the gold industries (**?**; **?**; **?**) Recent papers have studied commodity users, but only for small sectors. Examples are **?** and **?** for airlines and **?** for oil refineries. In our paper we extend the analysis and include all manufacturing sectors to study commodity shocks originating from a wide spectrum of commodities to study hedging in general equilibrium.[4]

Finally, there is an incipient literature on purchase obligations using the same database. **?** leverage on the introduction of steel futures in 2008 to show that purchase obligations are hedging instruments, because companies switch to these commodity futures once they become available. The authors use the same purchase obligations dataset we use, but include all sectors in the United States. Our paper considers only manufacturing industries because commodity price shocks are more important in these sectors than in services. On the other hand, **?** find empirical evidence showing that firms that hedge using purchase obligations can offset financial constraints and increase investment. We extend their work in three dimensions. First, we provide an estimate of the contribution of purchase obligations as a hedging device for material input price risk. Second, we build a general equilibrium model in which firms endogenously choose their supply contracts to reduce their exposure to input price volatility. Third, we quantify how these risk- management tools can dampen the transmission of input price shocks to aggregate variables.

Layout. The rest of the paper is organized as follows. Section 2 explains the data used in the paper. Section 3 discusses our empirical findings. Section 4 introduces the general equilibrium model of purchase obligations and studies its predictions. Section 5 quantitatively analyzes how the use of purchase obligations can dampen the propagation of input price shocks, and Section 6 concludes.

[3] For example, **?**, **?** and **?**.

[4] There is a vast literature regarding exchange rate hedging, for instance **?**, **?** , **?** , **?**, **?**, **?** and **?**.

2 Data and Background

Our main contribution is to describe how risk-management policies can have implications for the transmission of material input price shocks. We rely on a novel dataset on purchase obligations from U.S. public companies in the manufacturing sector. We also construct a sector input price index using the Bureau of Labor Statistics (BLS) and the 2012 Economic Census.[5]

Firms. We use COMPUSTAT for firm characteristics from public corporations in the U.S. manufacturing sector between 2003 and 2018.[6] We include purchase obligations for future periods from Securities and Exchange Commission (SEC) year filings. Companies are required to report their future obligations in their annual reports (10-Ks) under the Sarbanes-Oxley Act of 2002.[7][8] According to the SEC, a purchase obligation (PO) is a binding obligation for the purchase of goods and services that requires future payments and has fixed or variable quantities and prices.[9] For instance, a tire manufacturer could sign a contract with a supplier for the purchase of natural rubber. We use a Python code to download the relevant information described in the financial section of each firm's annual report.[10] Figure 2.1 shows a sample for the Cooper Tire & Rubber Company for their 2018 annual report. We have information on the total value of PO for future periods. We focus on future contracts with a maturity of 1 year. Companies briefly describe the content of their POs in a footnote. We manually reviewed about 1% of the sample to verify that companies include raw materials in their PO.

Some firms also report, in the text of their annual reports, the reason for using purchase obligations. For example, The Hershey Company (a food manufacturing company) provides details in their 10-K for 2010. This is shown in Figure 2.2. We see that the main reason the company engages in these contracts is to reduce their exposure to input price risk.

The final product is a firm-level yearly dataset for all public manufacturing companies in the U.S between 2003 and 2018. Summary statistics are shown in Table 2.1. We observe about 935 companies per year in 21 industries within the manufacturing sector. A key empirical

[5]See Appendix B for more details.

[6]We do not include corporations with negative net income for all sample periods; profits from these companies might be unrelated to commodity price risk.

[7]For more details on the institutional background, see **?**.

[8]Smaller reporting companies are not required to disclose their purchase obligations. We treat company-year observations as missing if a company does not report purchase obligations.

[9]See https://www.sec.gov/rules/final/33-8182.htm.

[10]We thank Cando-IT for code support. Website https://candoit.com.ar/?lang=en.

Figure 2.1: Example Purchase Obligations: Tabular Disclosure

(Dollar amounts in thousands)	Payment Due by Period						
Contractual Obligations	Total	2019	2020	2021	2022	2023	After 2023
Unsecured notes	$ 290,458	$ 173,578	$ —	$ —	$ —	$ —	$ 116,880
Capital lease obligations and other	6,245	1,182	—	5,063	—	—	—
Interest on debt and capital lease obligations	88,150	23,127	9,241	8,994	8,912	8,912	28,964
Operating leases	125,729	31,711	27,861	17,158	12,951	9,324	26,724
Notes payable (a)	15,288	15,288	—	—	—	—	—
Purchase obligations (b)	308,812	253,967	54,845	—	—	—	—
Postretirement benefits other than pensions (c)	251,798	15,344	15,927	16,238	16,446	16,557	171,286
Pensions (d)	148,250	45,000	40,000	25,000	20,000	15,000	3,250
Income taxes payable (e)	20,145	—	1,372	—	2,614	7,181	8,978
Other obligations (f)	33,158	10,509	1,340	2,165	972	520	17,652
Total contractual cash obligations	$ 1,288,033	$ 569,706	$ 150,586	$ 74,618	$ 61,895	$ 57,494	$ 373,734

(a) Financing obtained from financial institutions in the PRC to support the Company's operations there.
(b) Purchase commitments for capital expenditures, TBR truck tires and raw materials, principally natural rubber, made in the ordinary course of business.
(c) Represents benefit payments for postretirement benefits other than pension liabilities.
(d) Represents Company contributions to retirement trusts based on current assumptions.
(e) Represents income taxes payable related to the deemed repatriation tax on undistributed earnings of foreign subsidiaries under the Tax Act, as based on the Company's most recently filed tax returns, as well as anticipated state income tax obligations.
(f) Includes stock-based liabilities, warranty reserve, deferred compensation, nonqualified benefit plans and other non-current liabilities.

Notes. The figure shows an extract of the disclosure of purchase obligations for Cooper Tire & Rubber Company in their annual report for 2018 (emphasis added). See `https://www.sec.gov/ix?doc=/Archives/edgar/data/0000024491/000002449119000012/a2018123110k.htm` page 33.

Figure 2.2: Example of Purchase Obligations: Cite Disclosure

Purchase Obligations

We enter into certain obligations for the purchase of raw materials. These obligations were primarily in the form of forward contracts for the purchase of raw materials from third-party brokers and dealers. These contracts minimize the effect of future price fluctuations by fixing the price of part or all of these purchase obligations. Total obligations for each year presented above consists of fixed price contracts for the purchase of commodities and unpriced contracts that were valued using market prices as of December 31, 2010.

Notes. Extract from The Hershey Company's 2010 annual report. See `https://www.sec.gov/Archives/edgar/data/47111/000119312511039789/d10k.htm` page 36.

Table 2.1: Dataset Summary Statistics

Panel A	**General Statistics**		
Total Obs	Total Sectors	Total Years	Firms (approx)
14,036	21	15	935
Panel B	**PO vs non-PO**		
Variable	PO > 0	PO = 0	
Mean Employment	11,448	5,987	
Mean Sales (M$)	5,440	1,911	
Firms %	66%	34%	
Panel C	**For PO users**		
Variable	Mean	Median	Std. dev.
PO (M$)	348	35	1,629
PO/COGS	13.2%	9.3%	13.7%

Notes. The table shows basic summary statistic for the dataset used in the paper.

finding is that firms that engage in risk-management policies are larger.

We can see this by comparing average sales and employment between hedgers and non-hedgers in Table 2.1 (Panel B). This can also be seen in Figure 2.4a. We group observations in bin deciles according to the sector-year distribution of sales. We compute the mean and median value of a PO indicator to measure the likelihood of firms in that sales bin to use purchase obligations. We see that the probability of engaging in risk-management policies is increasing in firm size. Small firms rarely use purchase obligations, whereas most large firms use them consistently over time and across sectors.

Along the intensive margin, the results are similar. Figure 2.4b shows the distribution of the natural logarithm of purchase obligations value across observations within the same bin of sales. The dataset suggests that purchase obligations are increasing in firm size, which shows that large corporations have stronger incentives to trade these contracts. When adjusting by cost of goods sold (see Figure 2.4c), we see that purchase obligations are roughly constant along the firm size. This is consistent with **?** as we are using the same database.

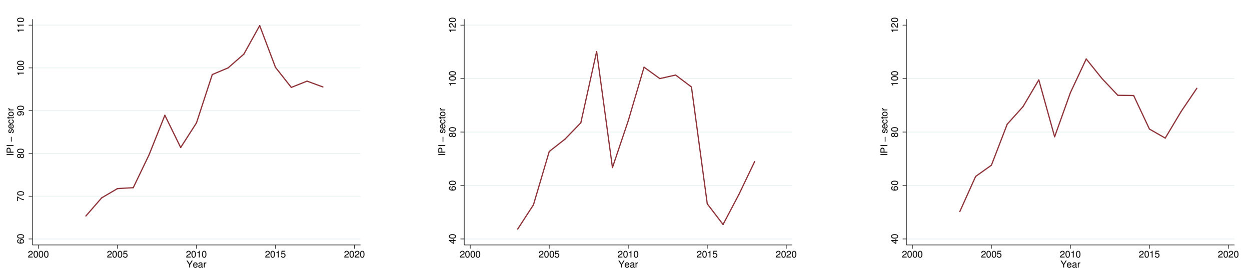
The dataset includes rich sector heterogeneity, as shown in Figure A.1. For each sector, we plot the distribution over time of the percentage of total purchase obligations over total material inputs demand. We use one-period-lagged PO values to capture a measure of sector hedging intensity. Median values of this distribution lie between 1% and 10% percent. The three sectors with more hedging intensity, according to this metric are Petroleum and coal products, Chemical products and Primary metals.

Limitations of the database include our inability to identify the share of the total value of purchase obligations that has fixed prices. For the empirics of the paper, we assume that the mayority have fixed prices. Another assumption in line with this argument is to treat the adjustment of PO prices as if they were less than perfectly colinear with spot prices. For example, if PO prices include an adjustment lag, they will provide insurance against commodity price shocks.

Input price index. It is well known in economics that commodities are several times more volatile than other goods and services. We constructed a sector-year input price index to measure the evolution of the commodity prices used in manufacturing. Material price indexes are from the BLS or World Bank. Using the Economic Census of 2012, we construct material purchases shares of each NAICS-3 manufacturing sector. The input price index (IPI) is a Laspeyres index using material shares and material price indexes. Table A.1 in the Appendix shows the most important material used in each sector (as share of total materials used). For instance, requirements include 9% cattle (Food Manufacturing, NAICS 311), 41% crude petroleum (Petroleum and Coal, NAICS 324), and 14% steel (Primary Metals, NAICS 331).

Figure 2.5 plots the evolution of our input price index for three representative industries. The plot shows large fluctuations of commodity prices. The estimated standard deviation of this period is 12%. This is common for all industries in our sample. We plot all sectors in Figure A.2 in the Appendix.

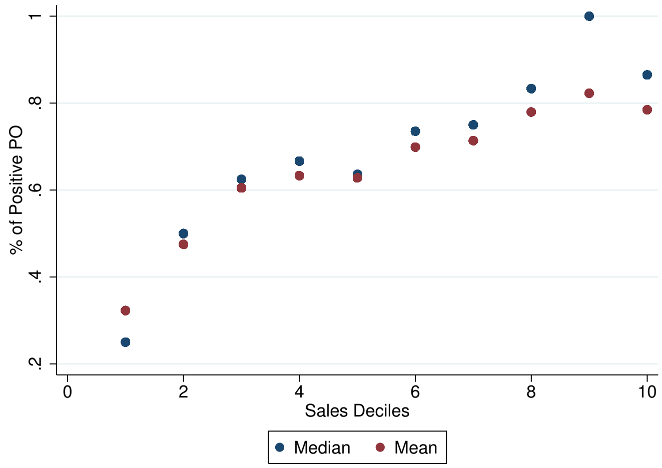
Figure 2.5: Input Price Index by sector



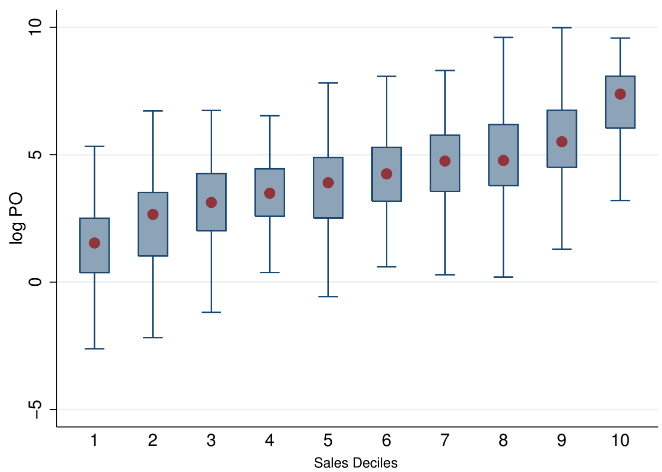
(a) Food Manufacturing (b) Petroleum and coal products (c) Primary metals

Notes. This figure shows the evolution of the input price index for a selected group of industries. It is constructed using material shares from the Economic Census 2012 for manufacturing sectors and commodity price indexes from the BLS and World Bank. Base year 2012.

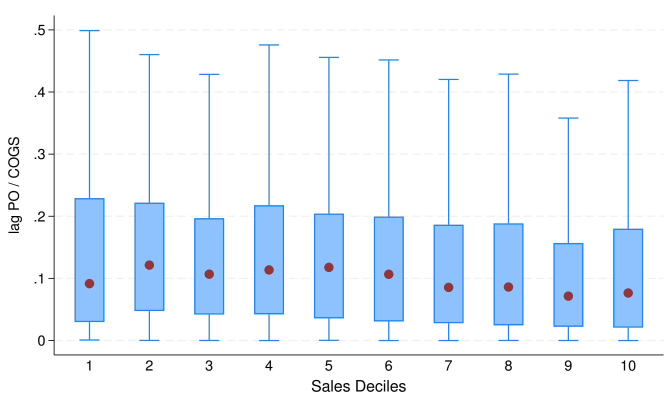
Figure 2.3: Purchase Obligations and Firm size



(a) PO - Extensive Margin



(b) PO - Intensive Margin



(c) PO/COGS - Intensive Margin

Notes. This figure show the distribution of purchase obligations across sales sales deciles. Panel (a) plots the mean and median of the purchase obligation indicator within bin. Panel (b) shows the distribution of the log PO across bin sales. Panel (c) shows the distribution of PO/COGS along bin sales.

3 Empirical Findings

In this section we discuss our two empirical findings. Purchase obligations are risk-management tools that allow firms to reduce their exposure to input price risk. We first show that firms that use these instruments have significantly lower exposure. The second finding suggests that these firm policies could dampen the aggregate transmission of input price shocks. We show that the correlation of input prices and sector aggregate variables is lower when firms trade more purchase obligations.

3.1 Risk-management Empirical Findings

We estimate how the use of purchase obligations can reduce firm exposure to input price risk. We follow the empirical literature in finance pioneered by **?** and **?**. The approach is to compute the elasticity of firm value with respect to the underlying risk. In the benchmark setting, we proxy for changes in firm value using stock returns. For the underlying risk, we use our measure of sector input price index computed using BLS and Census data.

We follow a log specification to estimate the elasticity using our dataset. The reduced-form equation will be:

$$\log\left(1+R_{i,s,t}\right)=\alpha+\beta_1\Delta IPI_{s,t}+\beta_2\Delta IPI_{s,t}*\mathbb{I}_{PO_{i,t-1}>0}+\epsilon_{i,t} \tag{1}$$

We define $R_{i,s,t}$ as the ex-dividend stock return of company i in sector s at time t and $\Delta IPI_{s,t}$ the change input price index (base 2012) of sector s between t and $t-1$. The key component of the regression is adding an indicator that takes the value of 1 if the company reported having positive purchase obligations in the previous year ($\mathbb{I}_{PO_{i,t-1}>0}$). The results are shown in Table 3.1. The first parameter, β_1, captures the semi-elasticity of returns (R) with respect to sectorial input prices (IPI) for non-PO users. The second parameter, β_2, is the most relevant for our paper because it captures the difference in semi-elasticity between companies that used purchase obligations in the past versus non-users.

Table 3.1: Input price elasticity estimation

	(1) log Returns	(2) log Returns	(3) log Returns
Change Sector IPI	-0.0138***	-0.0151***	-0.0148***
	(0.00168)	(0.00172)	(0.00184)
Change Sector IPI × lag Ind PO	0.00530**	0.00556**	0.00410+
	(0.00192)	(0.00195)	(0.00212)
Constant	0.0143**	0.0156***	0.0177***
	(0.00468)	(0.00461)	(0.00110)
Observations	13237	13237	13074
R^2	0.016	0.021	0.161
FE	None	NAICS 3	Firm

Notes. This table reports the estimation of input price elasticity using stock returns as a measure of change in firm value. The estimated equation is 1. Additional controls include fixed effects. Standard errors are clustered at the firm level and included in parenthesis. $^{+}$ $p < 0.10$, * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Using the third column, in which we include firm fixed effects, we find that that a 10% increase in input prices is associated with a decrease in returns of about 14.8% for firms that do not use purchase obligations. For companies that have supply contracts this number is only 10.7%. This stems from the fact that β_2 captures the differential effect.

This reduction is substantial: by computing the ratio of the coefficients, we obtain the percentage of the input price shock that hedgers can offset.[11] We also provide confidence intervals computed using the delta method.[12] Coefficients are shown in Table 3.2 using the coefficients for each log returns regression. Our preferred estimates are the coefficients estimated using the firm fixed effects specification. Based on our preferred estimates, PO firms have a differential exposure to input price changes of 27%. These numbers suggest that these supply contracts are an excellent tool to hedge against unexpected price changes in material costs.

[11] Specifically, $-\frac{\beta_2}{\beta_1}$.

[12] The delta method uses the Central Limit Theorem to compute the asymptotic distribution of a function of a random variable with known asymptotic distribution. See **?** chapter 5.

Table 3.2: Percentage of input-price shock hedged - log Returns

Model	Point Estimate	S.E.	Conf. Int. L.B.	Conf. Int. U.B.
OLS	38.3	18.11	20.19	56.41
FE Sector	36.91	16.73	20.19	53.64
FE Firm	27.76	17.48	10.28	45.24

Notes. This table reports a summary of the log returns estimations. We report the share hedged as the (negative) ratio of the estimated coefficients and confidence intervals computed using the delta method.

3.2 Robustness

We conduct a series of robustness checks to provide further evidence supporting our hypothesis. First, we use other measures of firm value to estimate the differential exposure to input price risk. Second, we estimate different coefficients along the distribution of the dollar value of purchase obligations. Finally, we allow differential coefficients for increases or decreases of input prices.

Other measures of firm value. We introduce other measures of firm value: net income (NI), earnings before taxes ($EBIT$), and earnings before taxes and depreciation ($EBITDA$). We normalize by total assets and estimate the reduced form:

$$\Delta \left(\frac{V_{i,t}}{AT_{i,t}} \right) = \alpha + \beta_1 \Delta IPI_{s,t} + \beta_2 \Delta IPI_{s,t} * \mathbb{I}_{PO_{i,t-1}>0} + \epsilon_{i,t} \tag{2}$$

The results are shown in Table A.3. Fixed effects are included in Table A.4. Our hypothesis remain consistent for these other firm value measures: Firms that use purchase obligations have lower exposure to input price risk. We plot the implied input price elasticity in Figure A.3 using the delta method. For NI, $EBIT$, and $EBITDA$, we normalize by the median ratio of firm value to total assets. All plots show that firms that use purchase obligations have a lower input price elasticity.

Controlling for hedging intensity. We use the dollar amount of purchase obligations to estimate heterogeneous coefficients across firms. We measure the intensity of hedging by computing the ratio of PO value chosen in the previous period and cost of goods sold ($COGS$). We repeat the estimation strategy using our measures of firm value, but include this new hedging intensity variable. The reduced form is:

$$X_{i,t} = \alpha + \beta_1 \Delta IPI_{s,t} + \beta_2 \Delta IPI_{s,t} \frac{PO_{i,t-1}}{COGS_{i,t}} + \epsilon_{i,t} \tag{3}$$

For X : $\log(1+R)$, $\Delta\ NI/AT$, $\Delta\ EBIT/AT$ or $\Delta\ EBITDA/AT$. The results are shown in Tables A.5, A.6, A.7 and A.8. Our estimates are still consistent with a positive exposure differential for firms using purchase obligations.

To highlight the intuition, we plot the implied elasticity of firm value to input price for several values of the distribution of PO/COGS. Delta method standard errors are included as confidence bands. The plots, which are presented in Figure A.4, show that purchase obligations are correlated with lower exposure to input price risk, but only if the firm has large contracts: Only firms above the median of the PO/COGS have lower exposure to input price risk.

Ups and downs. We test whether there are heterogeneous effects from increases or decreases of input prices. If firms have a large share of their input prices fixed due to purchase obligations, a decrease in the price should yield more benefit for corporations that do not use these contracts. These firms will have the flexibility to take advantage of the reduction in input prices. We estimate the following reduced form:

$$\begin{aligned}\log(1+R_{i,t}) = \alpha &+ \beta_1\ \Delta IPI_{st} + \beta_2\ \Delta IPI_{st} \times \mathbb{I}_{PO_{i,t-1}} \\ &+ \beta_3\ \Delta IPI^{+}_{st} + \beta_4\ \Delta IPI^{+}_{st} \times \mathbb{I}_{PO_{i,t-1}} + \epsilon_{it}\end{aligned} \tag{4}$$

To measure differential effects from ups or downs in input prices, we construct $\Delta IPI^{+}_{st} = \Delta IPI_{st} * \mathbb{I}_{\Delta IPI_{st}>0}$. Therefore, β_1, captures the correlation between firm value and input prices only for reductions and non-PO. On the other hand, $\beta_1 + \beta_3$ show the correlation for price increases. Finally, the key coefficients in this reduced form are β_2 and $\beta_2 + \beta_4$, which capture the differential effect for PO firms.

We include these results in Table A.9. Basically, our results show that the use of purchase obligations substantially benefits firms for positive input price shocks. Our estimates suggest that a decrease in input prices does not significantly benefit non-PO firms over PO firms.

3.3 Discussion on Endogeneity

The causal inference of the interaction parameter is guaranteed if there is no correlation between unobservables and the purchase obligations. The main problem is that firm size might be a confounding factor, as we saw above. We see this by estimating a linear probability model of the PO indicator on several standard hedging determinants (e.g., **?**). Results are shown in Table A.2. There is a positive correlation between purchase obligations and firm financial characteristics that will overestimate the hedging contribution.

We address this endogeneity concern by repeating the estimation of the input price elasticity using the ratio of PO/COGS and controlling for firm characteristics. Larger firms might have a lower input price elasticity than smaller firms. The estimated equation is:

$$\log(1+R_{i,s,t,}) = \alpha + \beta_1 \Delta IPI + \beta_2 \Delta IPI \times \frac{PO_{i,t-1}}{COGS_{i,t}} + \sum_l \beta_l \Delta IPI \times Control^l_{i,t-1} + \epsilon_{i,t} \tag{5}$$

We include controls that proxy for the firm's financial soundness and size, for example Working Capital/Total Assets, Retained Earnings/ Total Assets and log assets. The results are show in Table 3.3. The most important variable to control for the correlation between size and PO is total assets.

Our results show that larger firms are less exposed, regardless of their amount of purchase obligations, as seen in the estimated coefficient for the interaction of commodity prices and total assets. However, controlling for the commodity price risk exposure due to size, we still find a positive contribution of purchase obligations, as the interaction between commodity prices and PO/COGS is positive. The results are still consistent with our hypothesis: firms with a positive amount of purchase obligations have lower exposure to commodity price risk.

Table 3.3: Input-price elasticity estimation - controlling with assets

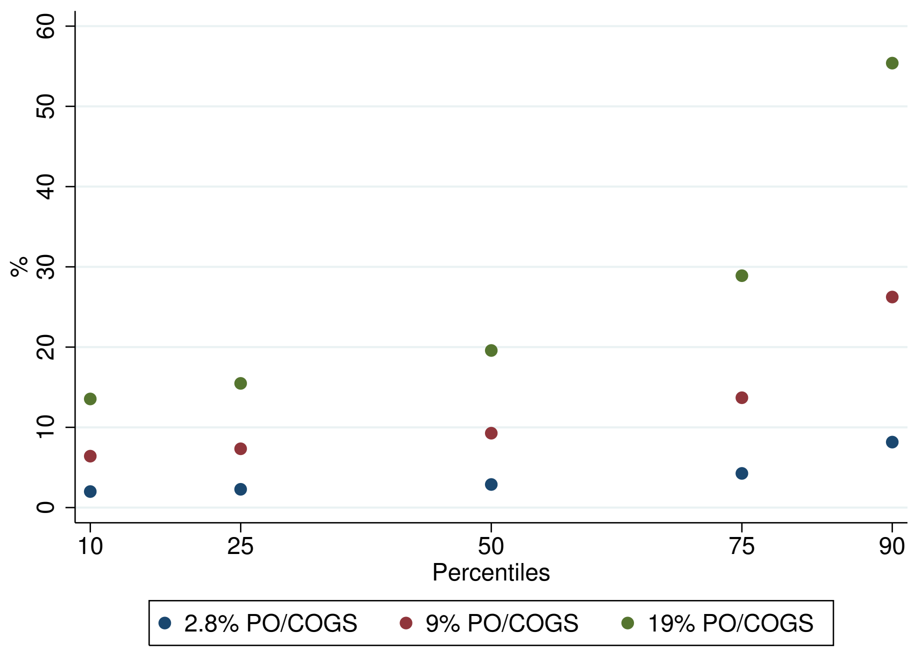
	(1) log Returns	(2) log Returns	(3) log Returns
Change Sector IPI	-0.0111^{***}	-0.0182^{***}	-0.0262^{***}
	(0.00109)	(0.00310)	(0.00451)
Change Sector IPI × lag PO/COGS	0.0169^{*}	0.0155^{*}	0.0132^{+}
	(0.00681)	(0.00664)	(0.00678)
Change Sector IPI × lag log Assets		0.00103^{**}	0.00149^{**}
		(0.000383)	(0.000468)
lag log AT		0.0208^{***}	0.0213^{***}
		(0.00256)	(0.00263)
lag PO / COGS		-0.0456	-0.0458
		(0.0382)	(0.0387)
Constant	0.0175^{***}	-0.116^{***}	-0.117^{***}
	(0.00465)	(0.0195)	(0.0199)
Other Controls	No	No	Yes
Observations	12989	12983	12639
R^2	0.014	0.021	0.022

Notes. This table reports the estimation of input-price elasticity allowing for different coefficients according to the hedging intensity and including additional firm controls. The estimated equation is 5. Additional controls include: Working Capital / Total Assets; Retained Earnings/ Total Assets; EBIT / Total Assets; Market Value of Equity / Book value of Liabilities; Sales / Total Assets. Standard errors are clustered at the firm level and included in parenthesis. $^{+}$ $p < 0.10$, * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

We study the implications of these results by computing the input price elasticity for different values of firm characteristics. For all calculations we use the estimated coefficients from Table 3.3 and different values of PO/COGS. We show summary statistics of the variables used in the estimation and analysis in Table A.10 and the results in Table A.11. We report a summarized version of the results in Figure 3.1, by computing the ratio of estimated elasticities between PO and non-PO firms, for different ratios PO/COGS and firms characteristics. This statistic represents the relative exposure between PO and non-PO firms.

Figure 3.1 shows several important conclusions. First, firms having positive PO are less exposed to commodity price shocks. For example, comparing two median firms where the PO firms has a 9% PO/COGS ratio, the exposure differential is about 10%. Second, the exposure differential is increasing in PO intensity and size. For instance, following the previous example, increasing the ratio PO/COGS to 19% we see that the exposure differential changes to about 20%. Moreover, computing the same statistic for firms at the top of the distribution (90% percentile), we find that PO firms are 27% less exposed when having a PO/COGS ratio of 9%. This statistic increases to 55% when using a ratio PO/COGS of 19%

Figure 3.1: Input Price Elasticity Estimation



Notes. This figure reports estimates of the input-price relative elasticity between PO and non-PO firms for different firm characteristics, following results from Table 3.3 (column 3). The x-axis corresponds to the percentiles of the size distribution according to the covariates used in the estimation. The y-axis corresponds to the percentage difference between PO and non-PO elasticity, for different PO/COGS ratios. Each color reports the relative elasticity comparing non-PO firms with PO-firms having the displayed PO/COGS ratio.

3.4 Aggregate Empirical Findings

The main hypothesis of our paper is that firms' risk-management policies could have consequences for the transmission of commodity price shocks to aggregate variables. This section shows evidence that supports this theory.

We first replicate the results by **?** for the United States to study the aggregate implications of commodity price shocks. Using their estimated SVAR model, we compute cumulative impulse responses for an increase in commodity prices. In their model they use three commodity price indexes: agriculture, fuels and metals from the World Bank. Their model also includes real interest rates, as these might affect intertemporal decisions, expanding their effects of commodity price shocks. In summary the estimated model is

$$\begin{bmatrix} p_t^a \\ p_t^f \\ p_t^m \\ r_t \\ Y_t \end{bmatrix} = F \begin{bmatrix} p_{t-1}^a \\ p_{t-1}^f \\ p_{t-1}^m \\ r_{t-1} \\ Y_{t-1} \end{bmatrix} + G \begin{bmatrix} \mu_t^a \\ \mu_t^f \\ \mu_t^m \\ \mu_t^r \\ \epsilon_t \end{bmatrix}$$

where p^a, p^f and p^m represent the price indexes (agriculture, fuels and metals), Y and r are real GDP and world real interest rates. All variables are constructed by detrending the natural logarithm using the HP filter (1600 smoothing parameter). Real commodity prices capture the cyclical component of real world prices (nominal index divided by U.S. Consumer price index). Real interest rates are computed using the difference between the U.S. Treasury bill rate and CPI inflation.

A key identification assumption is that commodity prices are not affected by contemporary RGDP innovations ϵ. Therefore, the model also assumes that RGDP lag does not affect future commodity prices. In matrix notation:

$$F = \begin{bmatrix} A & \emptyset \\ B & C \end{bmatrix} \qquad G = \begin{bmatrix} I & \emptyset \\ D & I \end{bmatrix}$$

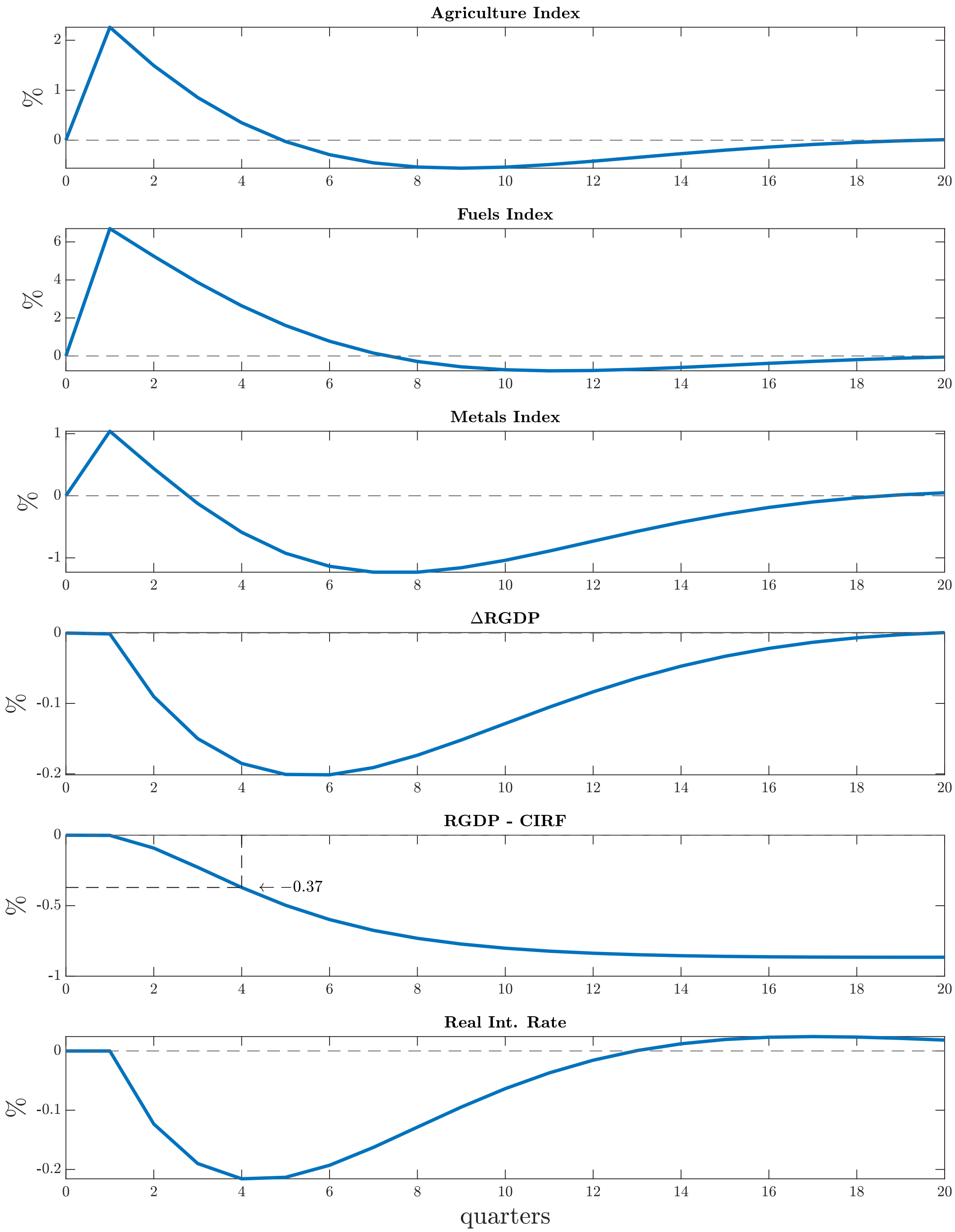
where A, B, C and D are estimated matrices and I is an identity matrix.

We combine the commodity prices using weights from World Bank to have a combined commodity price index.[13] We then proceed to compute the response of real GDP after a 10% increase in the index, constructed assuming a change in the shock parameters μ^j that yields a 10% change in the combined index. Results are showed in Figure 3.2.

The first three plots show the evolution of each component of the commodity price index. Notice that most of the index follows the movement of fuels. The fourth plot shows the quarterly change in real GDP. We see that it starts declining drastically one quarter after the shock. The fifth plot shows the cumulative evolution of RGDP. After four quarters, RGDP displays a decline of 0.37% compared to the pre-shock level. We will use this statistics in the calibration to match the aggregate movement of RGDP in the model. This is consistent with estimates by **?**.

[13] Specifically: 66.67% fuels, 22.83% agriculture and 10.5% metals.

Figure 3.2: Real GDP and Commodity Prices - U.S.

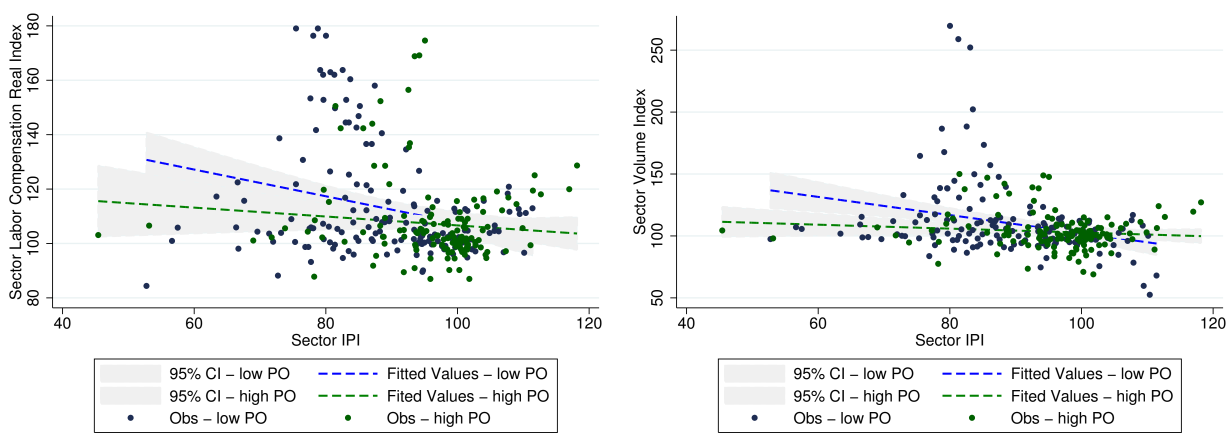


Notes. This plot shows impulse response functions of the estimated SVAR model assuming a 10% increase in the combined commodity price index.

Next, we show motivating evidence that highlights the contribution of purchase obligations on dampening the aggregate consequences of commodity price shocks. In Figure 3.3, we plot the Labor Compensation and Output Volume Index from the Bureau of Economic Analysis (BEA) as a function of our input price index for all sector-year observations in our sample. Both variables are measured as indexes with base in 2012. For Labor Compensation, we divide by the yearly Consumer Price Index from the BLS to measure real labor compensation. We split the sample according to the previous year's total purchase obligations of the sector, using the median value within sectors as the cutoff. Finally, we show the regression line for each group.

Consistent with previous studies (e.g., **?**) we found a negative correlation between labor compensation and input prices. However, these elasticities are smaller when firms in the sector have a total purchase obligation value over the median. This suggests that these risk-management policies could dampen the transmission of input price shocks to aggregate variables.

Figure 3.3: Sector Variables and Input Prices



(a) Sector Labor Compensation (b) Sector Volume Index

Notes. This figure shows the correlation between input prices and sector aggregate variables for different values of purchase obligations. Panel (a) shows sector labor compensation divided by CPI and Panel (b) shows sector volume. Both measures are expressed as indexes normalized to 2012.

4 A Model of Purchase Obligations

In this, section we introduce a general equilibrium model that explains why firms use purchase obligations and highlights the macroeconomic implications of commodity price hedging. We extend a heterogeneous firm trade model (?) without entry and exit, materials required for production, and sectoral heterogeneity along the lines of ?. The model will also feature financial constraints that will create incentives to hedge commodity price shocks using purchase obligations.

There will be heterogeneous monopolistic firms in two sectors, manufacturing and services that require materials, labor and capital to produce differentiated varieties. We use the notation i and $v = m, s$ to denote firm i belonging to sector v. There will be also be a representative firm in each sector that will bundle all varieties into a final sectoral good.

The model will have one time period and two stages. In the first stage, firms in the manufacturing sector set up their purchase obligation contracts. These are modeled as a supply contract for the delivery of a fixed quantity of material inputs at a fixed price (future price). Firms maximize expected profits conditional on the distribution of material prices in the second stage (spot prices). The main benefit of using purchase obligations is to ease the financial constraints present in the next stage.

In the second stage, all firms will make production and pricing decisions conditional on their productivity and the spot price, following a production function that requires labor, materials and capital. Firms in the manufacturing sector will have to finance their costs through borrowing. They could do this internally at zero interest rate, or externally at a positive interest rate. They main feature of the model is that firms will face a financial constraint that will limit their capacity for internal borrowing. Purchase obligations allow firms to reduce the negative consequences of commodity price shocks by easing the distress costs associated with the financial constraint.

To close the model, we assume a representative consumer/worker demanding goods from both sectors and supplying labor. All materials will be imported from the rest of the world and paid for using exports of the manufacturing bundle.

We include services in the model for the following reasons. First, we match the sectorial composition of developed economies: services represent a large share of GDP. In the data, firms in service sectors do not report using PO for material purchases. They contract on other services such as advertising or information technology. The main driver of this fact is that firms in the service sector require a low share of material inputs in production and are less exposed to commodity price risk. Secondly, the service sector will benefit from the PO decisions from manufacturing firms and this will increase the aggregate benefit of the use of purchase obligations.

4.1 Model Description

4.1.1 First Stage

Firms in manufacturing sector m can trade purchase obligations. Each firm i will choose its purchase obligations contracts quantity $\bar{m}_i^m$ by maximizing expected profits, where short positions are not allowed. Firm productivity z_i^m is constant and known in all stages. However, the spot price of raw material q is not observable at this stage. Firms will have to pay a purchase obligation price $\bar{q}$ for each unit of the future contract they purchase and face a contracting cost function $\kappa_b \bar{m}_i^m$ to access this market in addition to a fixed cost κ_a. We assume fair pricing by setting $\bar{q} = \mathbb{E}_q[q]$. Additionally, firms choose capital demand k_i^m facing rental rate r.

Firm's i problem in manufacturing results in

$$\max_{\bar{m}_i^m, k_i^m} \; \mathbb{E}_q[\Lambda(q)\pi_i^m(z_i^m, q, \bar{m}_i^m, k_i^m)] - \kappa_a - \kappa_b \bar{m}_i^m \quad \bar{m}_i^m > 0 \text{ and } k_i^m > 0, \tag{6}$$

where $\pi_i^m(z_i^m, q, \bar{m}_i^m, k_i^m)$ will be the firm's profit conditional on spot price q, and $\Lambda(q)$ is the stochastic discount factor of the representative consumer who owns all the firms. These functions will be derived in the next stage of the model.

For services, there will be a representative firm that chooses its capital demand in this first stage:

$$\max_{K^s} \; \mathbb{E}_q[\Lambda(q)\Pi^s(Z^s, q, K^s)] \quad K^s > 0, \tag{7}$$

where Z_s is the services productivity and K^s the capital choice. Productivity is constant and know in all stages.

4.1.2 Second Stage–Production

In the second stage, the spot price becomes public information. All firms make their production and pricing decisions. We first describe the production decisions of firms in the manufacturing sector. We assume there will be a unit mass of firms with cumulative density function H^m, where each firm behaves as a monopolist when selling its variety, but taking as given the price of the materials used in production, wages and rental rate. Materials can be purchased in two markets: a spot market and a purchase obligations market, with perfectly competitive prices q and $\bar{q}$. Let $\{m_i^m, \bar{m}_i^m, l_i^m\}$ be firm's i input choice: spot material purchase, purchase obligations material purchases, and labor demand. We assume the production function of firm i in manufacturing is[14]

$$y_i^m = z_i^m(\bar{m}_i^m + m_i^m)^{\gamma^m}(l_i^m)^{1-\gamma^m}(k_i^m)^{\frac{\varphi^m}{1-\varphi^m}}, \tag{8}$$

where z_i^m represents productivity and γ^m the materials share of production in the manufacturing sector. Notice that spot purchases m_i^m and PO purchases $\bar{m}_i^m$ are assumed to be perfect substitutes. Moreover, $\bar{m}_i^m$ is exogenous at this stage, since it was chosen in the first stage. Firms are allowed to sell their PO materials with no penalty. This implies that m_i^m can be negative. Capital chosen in the first stage k_i^m improves output capacity, where φ^m captures the marginal productivity of capital. The rental of capital is paid in this second stage therefore it is included in costs.

The cost minimization problem of firm i results in

$$\max_{m_i^m, l_i^m} \; qm_i^m + \bar{m}_i^m\bar{q} + wl_i^m + rk_i^m \text{ s.t. } z_i^m(\bar{m}_i^m + m_i^m)^{\gamma^m}(l_i^m)^{1-\gamma^m}(k_i^m)^{\frac{\varphi^m}{1-\varphi^m}} \geq y_i^m. \tag{9}$$

Proposition 1. *The solution of the problem above yields the following equations:*

$$cogs_i^m(w,q,\bar{q},r) = \frac{y_i^m}{z_i^m(k_i^m)^{\frac{\varphi^m}{1-\varphi^m}}}MC^m + \bar{m}_i^m[\bar{q} - q] + rk_i^m \tag{10}$$

$$MC^m(q,w,r) = \left(\frac{q}{\gamma^m}\right)^{\gamma^m}\left(\frac{w}{1-\gamma^m}\right)^{1-\gamma^m}$$

$$l_i^m(q,w,r) = \frac{y_i^m}{z_i^m(k_i^m)^{\frac{\varphi^m}{1-\varphi^m}}}MC^m\frac{1-\gamma^m}{w} \tag{11}$$

$$m_i^m(q,w,r) = \frac{y_i^m}{z_i^m(k_i^m)^{\frac{\varphi^m}{1-\varphi^m}}}MC^m\frac{\gamma^m}{q} - \bar{m}_i^m.$$

Proof. See Appendix E □

These represent total cost $cogs_i^m$, common marginal cost component for manufacturing firms MC^m, labor demand l_i^m, and spot material demand m_i^m. The critical deviation from a

[14]For tractability, we define production the production function as: $(y_i^m)^{1-\varphi^m} = (z_i^m)^{1-\varphi^m}(\bar{m}_i^m + m_i^m)^{\gamma^m(1-\varphi^m)}(l_i^m)^{(1-\gamma^m)(1-\varphi^m)}(k_i^m)^{\varphi^m}$. A simple algebra rearrangement yields the production function above.

standard Cobb-Douglas cost function is the second term in the cost equation. Because firms will have an "endowment" of materials coming from the purchase obligation market, they will receive an income effect that will change the effective total cost. This income effect will depend on the difference between the spot and the PO price.

For instance, if the PO price is higher than the spot price, firms will face higher total costs. Because there are no costs for selling the PO position, firms will completely sell out their PO materials at price q and repurchase the optimal quantity. The firm's financial gain or loss will depend on the price difference times the units traded: $\bar{m}_i^m[\bar{q} - q]$.

Firms in the service sector do not face financial constraints and cannot trade purchase obligations. We assume an analogous cost minimization problem as in manufacturing, with material cost share γ^s and capital productivity parameter φ^s. To keep the model tractable, we assume that all firms in this sector are identical, with productivity Z^s. The relevant equations for firms in this sector are

$$cogs^s(w,q) = \frac{y^s}{Z^s(K^s)^{\frac{\varphi^s}{1-\varphi^s}}} MC^s + rK^s \qquad MC^s(q,w) = \left(\frac{q}{\gamma^s}\right)^{\gamma^s} \left(\frac{w}{1-\gamma^s}\right)^{1-\gamma^s} \tag{12}$$

$$l^s(q,w) = \frac{y^s}{Z^s(K^s)^{\frac{\varphi^s}{1-\varphi^s}}} MC^s \frac{1-\gamma^s}{w} \qquad m^s(q,w) = \frac{y^s}{Z^s(K^s)^{\frac{\varphi^s}{1-\varphi^s}}} MC^s \frac{\gamma^s}{q}.$$

4.1.3 2nd Stage–Financing

Firms in the manufacturing sector borrow funds to cover their total costs $cogs_i^m$ from a domestic representative financial intermediary with deep pockets. They repay the loan plus interests in the same stage. Both actions take place after the uncertainty over the material input spot price has realized. We assume that the lender has deep pockets in the sense that it has unlimited amount of funds to lend. Borrowing firms face financial constraints, which we model as an interest-rate premium that inversely follows profits over costs. We build on **?** by assuming an elastic functional form for interest rates:[15]

$$1 + r_i^* = \left(\frac{\pi_i^m}{cogs_i^m}\right)^{-\iota}, \tag{13}$$

where π_i^m represents profits, $cogs_i^m$ total production costs, and $\iota > 0$. The ratio of profits over costs can be interpreted as the rate of return of the firm. They borrow $cogs_i^m$ to cover total costs, and receive profits π_i^m. If firms have low rate of return, the lender interprets this

[15]Technically we assume $1 + r_i^* = \exp\left\{-\iota\left[\log\left(\frac{\pi_i^m}{cogs_i^m}\right) - \log(1)\right]\right\}$. Minor algebra steps deliver the expression above.

as more risky and require a firm-specific interest-rate premium $r_i^* > 0$ to compensate for this additional risk. The paremeter ι captures the elasticity of the interest-rate premium relative to the rate of return.

This assumption follows **?** as we assume the firms require external financing at a higher cost than internal financing. As we will show below, this distress costs are increasing in spot prices because profits decline faster than costs when commodity prices increase. Therefore, the burden of external financing becomes more damaging for borrowers as input prices increase. An alternative interpretation of this constraint follows **?**. In that paper, lenders can only capture a fraction of profits, therefore they require collateral to proceed with the loan. In our paper, we can interpret r_i^* as the extra return the lender requires to compensate for the potential loss.

Let p_i^m be the price chosen by firm i and $y_{i,m}$ its product demand. Profits are

$$\pi_i^m = p_i^m y_i^m - \frac{y_i^m}{z_i^m (k_i^m)^{\frac{\varphi^m}{1-\varphi^m}}} MC^m(1+r_i^*) + \bar{m}_i^m[q-\bar{q}](1+r_i^*) - rk_i^m(1+r_i^*). \tag{14}$$

Purchase obligations consist of a supply contract that changes profits according to the difference between spot and future prices (see equation (14)). For large input prices ($q > \bar{q}$), the firm receives positive income and can reduce the interest premium charged by lenders

4.1.4 Second Stage–Firms' Profits

Firms will choose prices to maximize profits conditional on their variety demand. We assume there is a representative firm that aggregates all varieties within a sector, as in **?**. Elasticities of substitution are σ^m and σ^s for manufacturing and services, respectively, and both are larger than 1. Demand for variety i in sector $v = m, s$ is then $\left(\frac{p_i^v}{P^v}\right)^{-\sigma^v} Y^v$, where P^v and Y^v represent the price index and total demand for sector v.

The pricing decision for firms in the manufacturing industry will be the solution of the problem

$$\max_{p_i^m} \; p_i^m y_i^m + \left(\bar{m}_i[q-\bar{q}] - \frac{y_i^m}{z_i^m} MC^m(1+r_i^*) - rk_i^m\right)(1+r_i^*) \quad \text{s.t.} \quad y_i^m = \left(\frac{p_i^m}{P^m}\right)^{-\sigma^m} Y^m. \tag{15}$$

Proposition 2. *We characterize the pricing rule that maximizes profits, conditional on the interest rate:*

$$p_i^m = \frac{\sigma^m}{\sigma^m - 1} \frac{MC^m}{z_i^m (k_i^m)^{\frac{\varphi^m}{1-\varphi^m}}} (1 + r_i^*)(1 + \iota) \tag{16}$$

Proof. See Appendix E □

Firms will adjust their prices following the marginal costs adjusted by interest rates. A surge in input prices will generate two effects on prices. First, production costs will increase through the marginal cost and firms will raise prices. Second, as profits decrease, firms will face a larger interest-rate premium and further increase prices.

Proposition 3. *Using the optimal pricing rule, firms profits (conditional on financial constraints) are*

$$\pi_i^m = \left(\frac{\sigma^m - 1}{\sigma^m}\right)^{\sigma^m} (z_i^m)^{\sigma^m - 1} (k_i^m)^{(\sigma^m - 1)\frac{\varphi^m}{1-\varphi^m}} \left(\frac{q}{\gamma^m}\right)^{(1-\sigma^m)\gamma^m} \left(\frac{w}{1-\gamma^m}\right)^{(1-\sigma^m)(1-\gamma^m)} \times \tag{17}$$
$$(1 + r_i^*)^{1-\sigma^m} (P^m)^{\sigma^m} Y^m \left[\frac{1 + \iota\sigma^m}{\sigma^m - 1}\right] + (\bar{m}_i^m [q - \bar{q}] - r k_i^m)(1 + r_i^*),$$

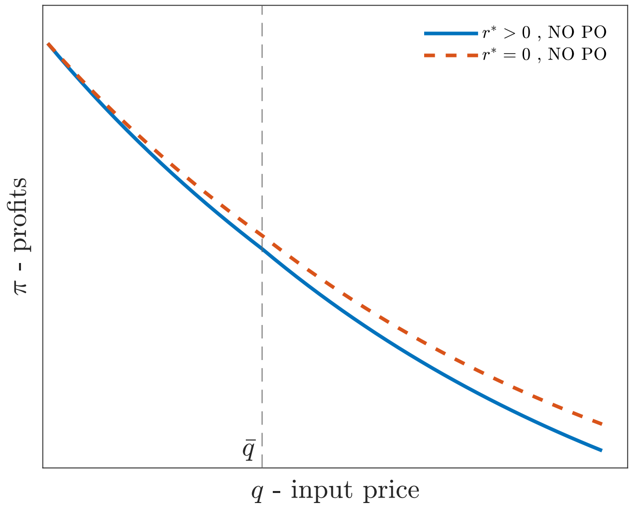
Proof. See Appendix E □

One one hand, profits are decreasing in spot prices though changes in the marginal cost and interest rates. On the other hand, they are proportional to the financial gain $\bar{m}_i^m[q - \bar{q}]$. For large spot prices, companies with purchase obligations receive positive income from the forward operation and can partially offset the reduction in sales from surges in input prices.

We compare profits when firms do not face a positive interest rate in Figure 4.1. The main impact of the external financing cost is that it drastically reduces profits when commodity prices increase. This is captured graphically by the difference between the two curves. Since firms earn less income due to increases in spot prices, they face tighter financial constraints (larger r^*)[16]. Specifically, Figure 4.1 plots log profits normalized to reduce level differences, conditional on the same aggregate variables (P_m, w, Y_m). The financial constraints create a level and a slope change in profits. We normalize profits in this plot to focus on the slope difference. An increase in spot prices reduces profits more if the firm faces financial constraints $(\iota > 0)$.

[16]The introduction of capital plays a crucial role as it is a fixed cost. This is because profits decline at the same rate as variable costs when spot prices increase. Therefore, profits decline faster that total costs when we include capital. Without capital, profits over total costs will remain constant when spot prices increase, hence financial constraints will also remain constant.

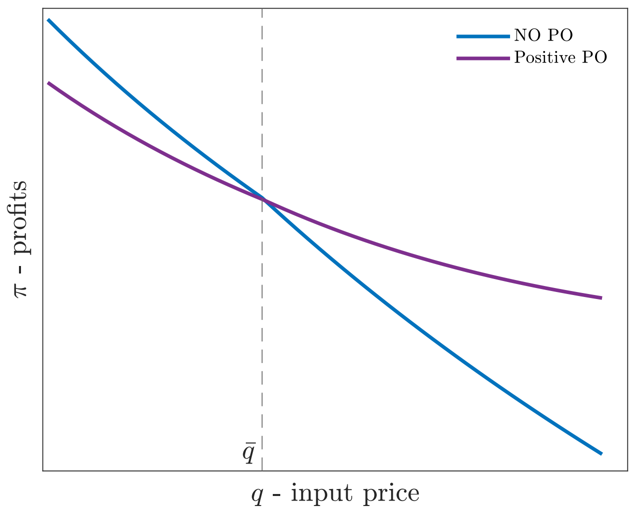
Figure 4.1: Firm's Profits–Zero Purchase Obligations



Notes. The figure shows profits log π as a function of spot prices q (normalized to eliminate level differences). The plot highlights the difference between profits when firms face financial constraints (the difference between dashed orange line and solid blue). We removed index i, m for exposition.

The main reason firms choose positive purchase obligations is to reduce the external financing burden. This is shown graphically in Figure 4.2. The introduction of purchase obligations makes two main contributions to firm profits. First, firms receive a negative income effect if spot prices are low. This happens because the financial hedge only protects the firm for large realizations of input prices. This is shown in the figure as the difference between the curves in the left region $q < \bar{q}$. Second, having purchase obligations increases financial income for large spot prices. This additional financial income increases profits and allows the firm to reduce interest expenses.

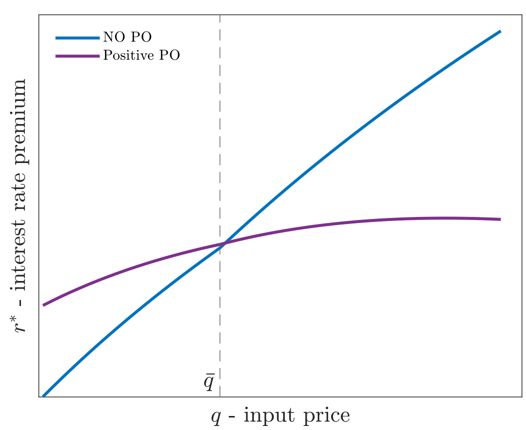
Figure 4.2: Firm's Profits–Adding Purchase Obligations



Notes. The figure shows the relationship between profits π and spot prices q and shows the benefit of contracting purchase obligations. We removed index i, m for exposition.

We plot the changes in interest rates when using purchase obligations in Figure 4.3. The interest rate is increasing in input prices because firms receive less income. When a firm trades purchase obligations, it will receive additional income that will generate a reduction in interest rates.

Figure 4.3: Interest Rates



Notes. The figure shows the relationship between interest rates r^* and spot prices q. Firms can reduce interest expenses by trading purchase obligations. We removed index i, m for exposition.

4.1.5 Second Stage–Consumer

To close the model, we include a representative consumer/worker and the rest of the world. We assume preferences as in **?** over hours and aggregate consumption, with relative risk aversion ρ. We also assume a Cobb-Douglas aggregator between sectors, with α being the share of income spent on manufacturing goods. Preferences are

$$U(C,L) = \frac{\left(C - \xi\frac{L^{1+\eta}}{1+\eta}\right)^{1-\rho} - 1}{1-\rho} \quad C = (C^m)^{\alpha}(C^s)^{1-\alpha} \quad C_v = \left(\int (c_i^v)^{\frac{\sigma_v - 1}{\sigma_v}} dH^v\right)^{\frac{\sigma^v}{\sigma^v - 1}} \quad v = s, m. \tag{18}$$

The budget constraint is $PC = wL + \Pi - \kappa^A + EF + rK$, where P stands for the final good price index, w for wages, Π for aggregate profits, L for hours worked, C for real consumption, κ^A for aggregate purchase obligations contracting costs, EF for all external financing costs paid by firms and rK income from renting capital (r is rental rate and K capital supply).[17] Consumers own all firms (manufacturing, services and financial intermediary), receive labor income wL and rental income as they also own the capital stock. Notice that the consumer will receive profits from all firms. This is important, because the hedging outcome will generate an income effect on consumers.

Given these assumptions, labor supply and goods demand are determined by the first-order conditions:

$$L = \left(\frac{w}{P\xi}\right)^{\frac{1}{\eta}} \qquad PC = wL + \Pi - \kappa^A + EF + rK \tag{19}$$
$$C^m P^m = \alpha PC \qquad C^s P^s = (1-\alpha)PC.$$

Definition 1 (External Financing)**.** External financing is defined as all borrowing costs paid by manufacturing firms to the representative lender

$$EF = \int_i \left(MC_m \frac{y_i^m}{z_i (k_i^m)^{\frac{\varphi^m}{1-\varphi^m}}} r_i^* - \bar{m}_i^m [q - \bar{q}] r_i^* + r k_i^m r_i^* \right) dH_m \tag{20}$$

Definition 2 (Auxiliar Aggregate productivity term)**.** Let Z_k^m be the interest-rate and capital adjusted productivity measure of manufacturing firms. Notice that this representation of aggregate productivity is tractable as financial constraints reduce the economies aggregate efficiency in manufacturing.[18]

[17] κ^A captures all PO negotiation costs: $\kappa^A \equiv \int_i \kappa_a \mathbb{I}_{\bar{m}_i^m > 0} dH^m + \int_i \kappa_b \bar{m}_i^m dH^m$., where $\mathbb{I}_{\bar{m}_i^m > 0}$ is and indicator that takes value of one if firm i has positive PO.

[18] To see this: $\frac{\partial\, Z_k^m}{\partial\, 1 + r_i^*} < 0 \; \forall \; i$

$$(Z_k^m)^{\sigma^m-1} \equiv \int \left(\frac{z_i^m (k_i^m)^{\frac{\varphi^m}{1-\varphi^m}}}{(1+r_i^*)(1+\iota)} \right)^{\sigma^m-1} dH^m$$

Using Definition 2, price indexes have a standard representation:

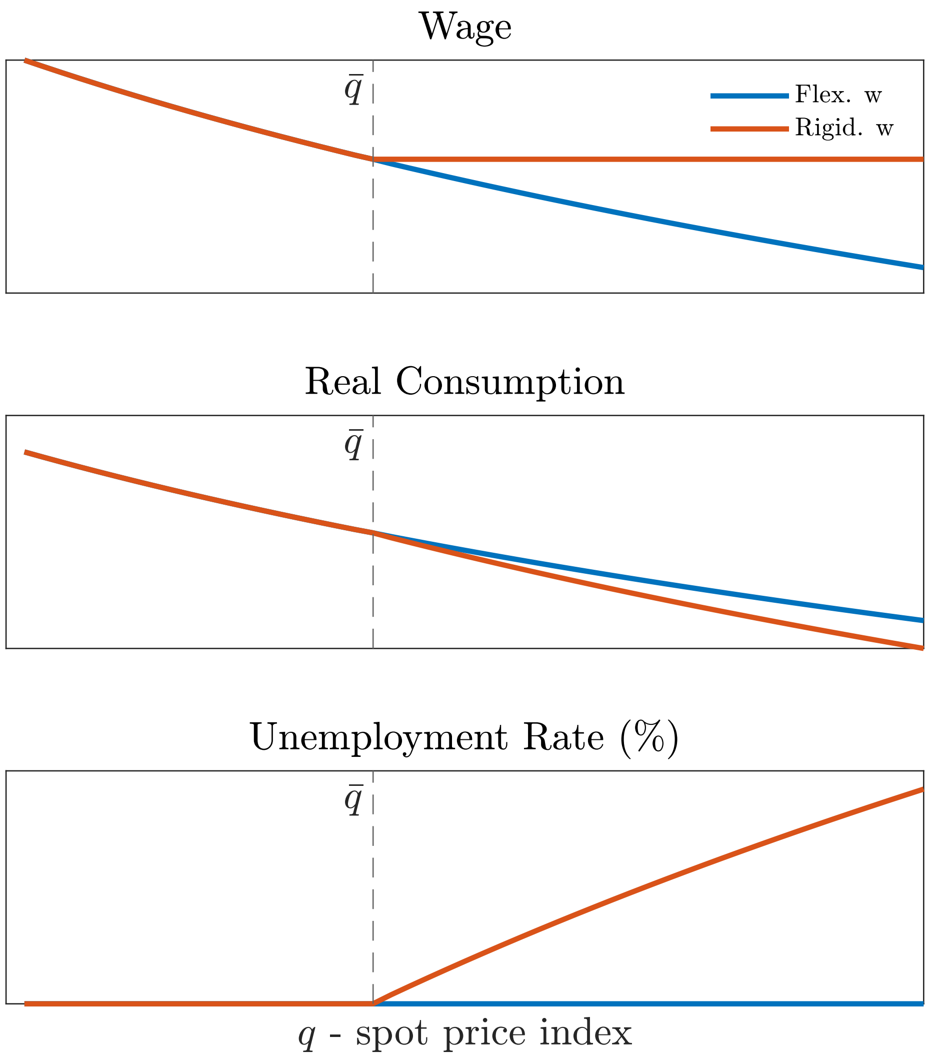
$$\begin{aligned} P &= \left(\frac{P^m}{\alpha}\right)^{\alpha} \left(\frac{P^s}{1-\alpha}\right)^{1-\alpha} \\ P^m &= \frac{\sigma^m}{\sigma^m-1} \left(\frac{q}{\gamma^m}\right)^{\gamma^m} \left(\frac{w}{1-\gamma^m}\right)^{1-\gamma^m} \frac{1}{Z_k^m} \\ P^s &= \frac{\sigma^s}{\sigma^s-1} \left(\frac{q}{\gamma^s}\right)^{\gamma^s} \left(\frac{w}{1-\gamma^s}\right)^{1-\gamma^s} \frac{1}{Z^s}. \end{aligned} \tag{21}$$

4.1.6 Second Stage–Downward Nominal Wage Rigidity

We assume that nominal wages face downward rigidities along the lines of **?**. In particular, we assume that wages are fixed if there is an increase in input prices, but they are flexible when input prices decrease, starting from mean spot prices. This is consistent with **?**, because the short-run response of wages to positive input price shocks is almost zero after the 2000s. This is also consistent with the asymmetric effects of commodity price shocks studied in the literature (e.g., **?**). In the quantitative exercises below, we study positive input price shocks, and therefore this assumption is relevant to match these empirical facts documented in the literature.

In general equilibrium there will be involuntary unemployment, denoted by u. In particular, u will be defined as the difference between labor supply and demand for fixed wage $\bar{w}$: $u(q, \bar{w}) = L^s(q, \bar{w}) - L^d(q, \bar{w})$. The downward rigidity implies that $u > 0$ when input prices increase, but $u = 0$ for decreases. Finally, $\bar{w}$ will be set as the flexible wage for the mean spot price $\mathbb{E}_q[q]$. For clarity, we plot the evolution of nominal wages, real consumption, and unemployment in Figure 4.4.

Figure 4.4: Downward Nominal Wage Rigidity



Notes. The figure shows the assumed relationship between nominal wages and spot prices. We also plot the model-implied relationship between spot prices and real consumption, and involuntary unemployment.

4.1.7 Second Stage–Foreign Economy

We assume two regions in the model: a domestic country and the rest of the world. It is easier to think about the domestic country as being the U.S., given our data on purchase obligations. Therefore, the domestic currency will be dollars. We assume the domestic country is a small open economy facing a perfectly elastic supply of material inputs (spot and futures).

The rest of the world will have preferences over the manufacturing domestic bundle and foreign final good, with ν being the elasticity of substitution and $1-\zeta$ its home bias:

$$C^* = \left((1-\zeta)^{\frac{1}{\nu}}(C^{m,*})^{\frac{\nu-1}{\nu}} + \zeta^{\frac{1}{\nu}}(C^{F,*})^{\frac{\nu-1}{\nu}}\right)^{\frac{\nu}{\nu-1}}. \tag{22}$$

The final good bundle will be exported with foreign demand $X = \left(\frac{\tilde{P}}{P^*}\right)^{-\nu}\zeta Y^*$, where P^* is the price index abroad, Y^* is real foreign income and $\tilde{P}$ is the price of the domestic final good good abroad. We assume that the law of one price holds in this economy: $\tilde{P} = PE$, where E is the exchange rate (measured as the purchasing power of domestic currency). We measure all variables in domestic currency (dollars) by setting the exchange rate to 1.

On the other hand, imports I will be all the materials needed for production measured in dollars. Since we solve for an balanced-trade equilibrium, the following equation must hold: $XP = I$. Therefore, the domestic final good price will adjust perfectly to reach balanced trade.

Definition 3 (Real GDP)**.** Finally, we define real GDP using base prices from mean spot price $\bar{q}$.

$$\text{real GDP}(q) = Y_s(q)P_s(\bar{q}) + Y_m(q)P_m(\bar{q}) - I(q)$$

4.2 Model Implications

In this section, we study the implications of the model. Subsection 4.2.1 defines an equilibrium and shows how distress costs increase the negative consequences of input price shocks. Subsection 4.2.2 shows how companies optimally engage in purchase obligations contracts.

4.2.1 Implications for Second Stage

Definition 4 describes a general equilibrium in the second stage.[19] Proposition 4 shows the relationship between distress costs and real wages in equilibrium and highlights the fact that distress costs increase the negative consequences of commodity price shocks.

Definition 4. A general equilibrium in this economy conditional on $(q, K^s, \bar{m}_i^m, k_i^m \ \forall \ i)$ for the second stage is defined as a set of aggregate prices (P, P^m, P^s, w) and firm choices $(p_i^v, m_i^v, l_i^v, y_i^v \ \forall \, i, v)$ and aggregate variables $(C, C^m, C^s, L, Y, Y^m, Y^s)$ such that

- Consumer is maximizing: Equation bloq (19)

[19] See Appendix C for a more detailed explanation of the algorithm.

- Firms are maximizing: Equations (18) and (13)
- Markets clear and balanced trade holds.

Proposition 4. *Changes in real wages from an increase in commodity prices (in constant wage region) can be expressed as:*

$$\mathrm{d}\log\frac{w}{P} = -\Gamma_q \mathrm{d}\log q + \alpha \mathrm{d}\log Z_k^m,$$

where $\Gamma_q = \gamma^m\alpha + \gamma^s(1-\alpha)$.

Proof. With fixed nominal wages $\mathrm{d}\log\frac{w}{P} = -\mathrm{d}\log P$. Using Equation bloq 21 and Definition 2, we arrive at the result. □

This proposition shows that an input price shock decreases real wages. However, due to the existence of distress costs, the impact of an input price shock can be larger depending on how interest rates react after the shock. This is captured by the reduction in measured productivity Z_k^m. The direct effect of input price shocks reduces profits for all firms, and hence interest rates increase. This further lowers wages, because these firms increase prices to offset the distress costs and reduce labor demand. Overall, the impact of the input price shock is larger.

4.2.2 Hedging and Capital demand determinants

Firms in manufacturing maximize expected profits conditional over the distribution of spot prices, conditional on negotiation costs. Purchase obligations can ease the financial constraints by reducing interest rates and increasing expected profits. Moreover, firms in both sectors choose their capital demand for all spot prices conditional on the rental rate.

Manufacturing firm i's problem in the first stage is

$$\max_{\bar{m}_i^m, k_i^m} \mathbb{E}_q[\Lambda(q)\pi_i^m(z_i^m, q, \bar{m}_i^m, k_i^m)] - \kappa_a \mathbb{I}_{\bar{m}_i^m>0} - \kappa_b \bar{m}_i^m \quad s.t. \quad \bar{m}_i^m > 0,$$

were $\Lambda(q)$ is the stochastic discount factor (defined below), π_i^m profits, κ_a fixed costs and κ_b variable costs of contracting PO.

Definition 5 (Stochastic Discount Factor)**.** Let the stochastic discount factor (SDF) in this economy be defined as

$$\Lambda(q) = \frac{\frac{\partial U(q)}{\partial C}/P(q)}{\frac{\partial U(\bar{q})}{\partial C}/P(\bar{q})}, \text{ where } \frac{\partial U}{\partial C} = \left[C - \xi \frac{L^{1+\eta}}{1+\eta}\right]^{-\rho}. \tag{23}$$

This is to correctly account for price changes in the discount factor and the relative scarcity towards the mean spot price $\bar{q}$ (starting point for shocks).[20]

Assuming that firms are small enough and do not internalize the aggregate effects of hedging, the first-order conditions are[21]

$$(\bar{m}_i^m): \quad \mathbb{E}_q\left[\Lambda(q)\frac{\partial \pi_i^m}{\partial \bar{m}_i^m}\right] \leq \kappa_b \quad \text{with equality if } \bar{m}_i^m > 0 \tag{24}$$

$$(k_i^m): \quad \mathbb{E}_q\left[\Lambda(q)\frac{\partial \pi_i^m}{\partial k_i^m}\right] = r. \tag{25}$$

Purchase Obligations choices.

Proposition 5. *We characterize the partial derivative for PO using the definition of profits:*

$$\frac{\partial \pi_i^m(q)}{\partial \bar{m}_i^m} = [q - \bar{q}] \times \underbrace{\frac{(1+r_i^*)\left[1 + \iota + \iota(\sigma^m - 1)\left[1 + \frac{\pi_i^m}{cogs_i^m(1+r_i^*)}\right]\right]}{1 + \iota + \iota\sigma^m\left[1 + \frac{\pi_i^m}{cogs_i^m(1+r_i^*)}\right]\left[\frac{\bar{m}_i^m}{\pi_i^m}[q-\bar{q}] - r\frac{k_i^m}{\pi_i^m}]\right]}}_{\text{Financial Accelerator}}$$

Proof. See Appendix E ☐

We plot the components of the first order condition in Figure 4.5. We subtract the income effect from PO $(q - \bar{q})$ and plot the financial accelerator and SDF. The black dotted line captures this normalization. All other lines can be interpreted as the difference in the first order condition relative to PO income for each spot price.

The blue line shows only the financial accelerator. Due to changes in financial constraints r^* the financial accelerator shows that the firm values more positive income received for large spot prices $(q > \bar{q})$ as this extra income can ease the constraints (reduce r^*). On the other hand, for $q < \bar{q}$, the negative income effect increases the financial burden of these constraints. Therefore, the firm reduces the value of these negative income.

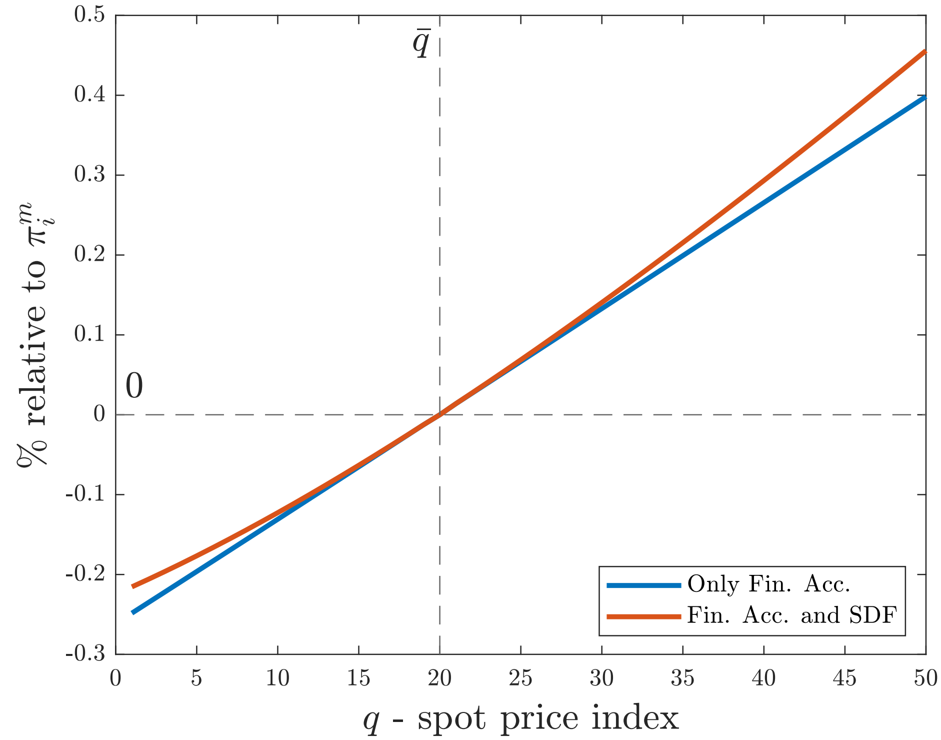
In red, we plot the combined component of financial accelerator and SDF. As the consumer values a smooth consumption path over spot prices, the firm values more the positive income

[20] See Appendix E for details.

[21] Specifically we are assuming $\frac{\partial \Lambda}{\partial \bar{m}_i^m} = \frac{\partial \Lambda}{\partial k_i^m} = 0 \; \forall \; i$.

effect for large spot prices, as it can deliver higher profits and consumption. When spot prices are low, the stochastic discount factor reduce the value of the negative PO income because the consumer already has relative high consumption.

Figure 4.5: First Order PO comparison



Notes. The figure shows the components of the first order condition for $\bar{m}^m$, where we subtracted the PO income $q - \bar{q}$. The blue line represents the additional income due to change in financial constraints (only financial accelerator). The red line shows the full first order condition including both the financial accelerator and the stochastic discount factor.

Services. For services the problem results

$$\max_{K^s} \; \mathbb{E}_q[\Lambda(q)\Pi_i^s(Z^s, q, k_i^m)] \quad s.t. \quad K^s > 0,$$

The solution implies

$$K^s = \left\{ \frac{1}{r}\mathbb{E}_q \left[\Lambda(q) \left(\frac{\sigma^s - 1}{\sigma^s} \right)^{\sigma^s} \frac{1}{\sigma^s - 1} (MC^s)^{1-\sigma_m} (Z^s)^{\sigma^s - 1} \frac{\varphi^s}{1 - \varphi^s} (\sigma^s - 1)(P^s)^{\sigma_s} Y^s \right] \right\}^{\frac{1-\varphi^s}{1-\varphi^s \sigma^s}}$$

4.3 Equilibrium First Stage

In the first stage of the model, all firms choose capital demand and manufacturing firms also set up their purchase obligations contracts as described above. The equilibrium in this stage is defined as a rental rate r where capital demand matches capital supply $\bar{K}$ (exogenously determined).

Definition 6. A general equilibrium in this economy for the first state is defined as a rental rate r such that

- Firms are choosing optimally their PO and capital
- Capital market clear: $\int_i k_i^m dH^m + K^s = \bar{K}$
- Second Stage Equilibrium conditions hold (see Definition 4)

5 Quantitative Exercises

We explore a series of quantitative exercises to understand the role of purchase obligations in the transmission of input price shocks. The main role of purchase obligations is to reduce the distress costs paid by firms when input prices increase. This is done by increasing income enough such that firms can reduce interest-rate expenses. In the aggregate economy, we find that the use of these contracts can provide insurance on input price shocks and dampen the aggregate transmission.

5.1 A stylized example

This section performs a simple counterfactual to provide insight into the main role of purchase obligations in the transmission of input price shocks. Imagine that spot prices start at a low value, with no purchase obligations. The economy faces an input price shock whereby the new spot price increases and all firms become more constrained (increase in $r_i^* \ \forall \ i$). With fixed nominal wages Proposition 4 solves for the response of real wages.

Proposition 6. *The solution for the change in equilibrium real wage can be written as*

$$
\begin{aligned}
\mathrm{d}\log\frac{w}{P} \approx & -\underbrace{\Gamma_q \mathrm{d}\log q}_{\text{direct effect}} \\
& \underbrace{-\,\alpha\int_i \frac{\left(z_i^m (k_i^m)^{\frac{\varphi^m}{1-\varphi^m}}(1+\iota)^{-1}(1+r_i^*)^{-1}\right)^{\sigma_m-1} dH^m(z_i^m)}{Z_k^{\sigma^m-1}} \mathrm{d}\log(1+r_i^*)}_{\text{distress costs}} .
\end{aligned}
$$

Proof. See Appendix E □

Two main effects explain the negative impact of input price shocks on real wages. On one hand, the shock affects aggregate variables directly by increasing input costs: Firms lower labor demand and increase prices due to the cost change. On the other hand, the increase in input costs reduces profits, and all firms become more constrained. Notice that the aggregate productivity measure declines due to the increase in interest rates. These are indirect costs, since firms now have to pay a larger interest rate on borrowed funds. As firms further raise prices and reduce labor demand, the equilibrium wage decreases further due to distress costs.

To study the role of purchase obligations, imagine a counterfactual situation in which all firms had a positive amount of these contracts that allowed them to avoid this constrained increase after the shock. An increase in the spot price will generate positive revenues from the hedge. In this counterfactual, we are assuming that this extra revenue will be large enough that none of the firms in the economy will face larger borrowing costs ($\mathrm{d}\log(1+r_i^*) = 0\ \forall$ i). Therefore, the wage response to an input price shock is lower.

$$
\mathrm{d}\log\left(\frac{w}{P}\right)^{PO} = -\Gamma_q \mathrm{d}\log q.
$$

As firms reduce their exposure to the shock, this provides insurance for the economy by reducing distress costs.

5.2 The role of purchase obligations

In this section we quantitatively study the contribution of purchase obligations to the transmission of input price shocks. We first discuss the calibration and then turn to several quantitative exercises. In particular, we compare the calibrated economy with an equilibrium in which firms cannot trade purchase obligations. We show that purchase obligations improve welfare and dampen the transmission of commodity price shocks.[22]

[22] See Appendix D for the algorithm used to solve the model.

5.2.1 Calibration

Parameters were chosen to match stylized facts of the U.S. economy and are shown in Table 5.1. Overall, the calibration follows targeted moments (see Table 5.2).

Table 5.1: Calibration

Parameter	Role	Value	Moment
$G(q)$	distribution spot prices	$\log \mathcal{N}(4.454, 0.082^2)$	IPI empirical distribution
$H^m(z)$	distribution of productivities	$\log \mathcal{N}(-1, 1.7)$	employment distribution manufacturing 2012
Z^s	productivity services	0.1	relative size manuf./services
γ^m	share of materials on cost (manuf.)	0.15	Empirical elasticity RGDP and materials
σ^m	elasticity of substitution (manuf.)	3	Avg. Markup Manuf.
γ^s	share of materials on cost (services)	0	normalization
σ^s	elasticity of substitution	1.65	Profit share services
φ^m	capital productivity in manuf.	0.147	Fixed Assets in Manuf. / sector GDP (BEA)
φ^s	capital productivity in services	0.48	Fixed Assets in Services / sector GDP (BEA)
$\bar{K}$	capital supply	78.90	Capital Income / GDP
α	share manuf. /GDP	0.13	Share Manuf./GDP
ι	interest-rate elasticity	0.02	Avg. External Financing
$\bar{q}$	PO unit price	$\mathbb{E}[q]$	efficient markets
κ_a	PO negotiation cost (fixed)	7e-08	Measure PO firms
κ_b	PO negotiation cost (variable)	0.027	Relative materials elasticity (PO vs non-PO)
ν	export elasticity	2	?
η	inv. Frisch elasticity	2	?
ρ	relative risk aversion	2	?
ξ	level employment	15.45	Employment Manuf. 2012
P^*	foreign price level	1	normalization
Y^*	foreign real output	58.97	World GDP
ζ	foreign bias	0.4	Auclert et al (2022)

Manufacturing and Services Sectors. We choose γ^m to match the aggregate response of RGDP in the model with the SVAR estimated in the empirical section. For the elasticity of substitution across varieties σ^m we use the statistics from the manufacturing sector from **?**, who construct an average elasticity of substitution by sector (NAICS 3) using total sales and total costs from the Economic Census 2012. We choose σ^m in the model to match the weighted average markup for the manufacturing sector, using each sector's gross output from the BEA as weights. Finally, we chose α to match the share of consumption expenditures on manufactures for 2012. For Services, we choose σ^s to match the average profit share over time of all sectors without considering manufacturing industries. The relative employment of services and manufacturing is captured by the parameter Z^s. The share of material inputs in costs γ^s was chosen to be zero to highlight the fact that services does not rely heavily on materials.

Table 5.2: Model Fit

Moment	Data	Model
Empirical elasticity RGDP and materials	-0.28%	-0.37%
Avg. Consumption Exp. in Manuf. products	13%	13%
Avg. Markup	1.53	1.57
Relative materials elasticity (PO vs non-PO)	27%	20.97%
Measure PO Firms 2012	0.5%	0.48%
Ratio Services/Manuf. Gross Emp. 2012	7.55	4.89
External Financing Cost	2.4%	2.41%
Gross Output Manuf. 2012	5.77	5.9
Unemployment Manuf. 2012	7.3%	7.55%
Fixed Assets Services / Sector GDP	1.95	1.91
Fixed Assets Manuf. / Sector GDP	0.6	0.6
Rental Income / GDP	0.33	0.33

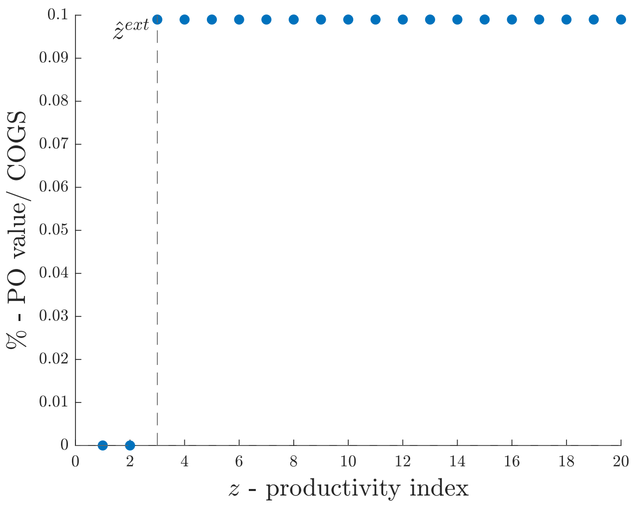
We match the distribution of spot prices and productivity in the model with the distribution of our measures of the input price index and employment in manufacturing for 2012. In particular, we set the mean of the spot price distribution so that gross output in the model matches manufacturing gross output in 2012. For labor parameters we set ξ to match the level of employment in manufacturing in 2012, assuming in that in full employment, the representative consumer spends one-third of her time working. Finally, we take the inverse Frisch elasticity η and risk aversion ρ from the macro-finance literature (**?** and **?**).

Financial Constraints and capital parameters. We calibrate financial financial constraints parameter ι to match the average external financing cost in **?**. For capital productivity parameters (φ^m and φ^s) we match the average ratio of fixed assets to sector GDP from BEA. For capital supply, we match the rental income to one third of GDP.

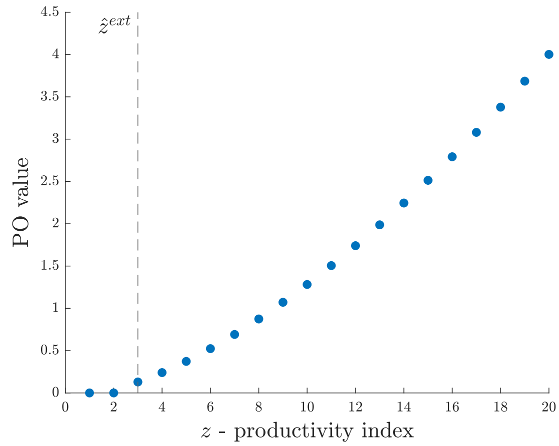
Purchase Obligations. The parameter κ_a allows us to match the share of firms that engage in purchase obligations. In the data of public firms, $\frac{1}{3}$ report having PO. As only 0.8% of the firms are public in the US (using Business Dynamics Statistics), we assume the share of PO is 0.53%. For κ_b we match the relative elasticity of returns to materials between PO and non-PO from the empirical analysis.

Rest of the world. Real GDP for the rest of the world Y^* is chosen to match the world GDP (net of US) in 2012. We take the remaining parameters from the literature. In particular, we take trade elasticity ν from **?** and foreign bias ζ from **?**

Figure 5.1: Model-implied Purchase Obligations and Firm size



(a) PO/COGS



(b) PO value

Notes. The figure shows the optimal purchase obligation choice along the firm-size distribution. Panel (a) normalize the coefficients by COGS in the model and Panel (b) shows the dollar amount of PO in the model.

5.2.2 Firm-size distribution

In this section, we study the quantitative predictions of the model in terms of the purchase obligation choice over the firm-size distribution. In Figure 5.2a we present the optimal purchase obligations value as a share of COGS for mean spot prices across the firm-size distribution.[23] Purchase obligations value is the product of the supply contract quantity and price: $\bar{m}_i^m \bar{q}$. In Figure A.5, we include the distribution of PO over COGS across the productivity and spot price distribution.

The figure shows that only large firms will choose positive purchase obligations. Also, the purchase obligation value is increasing in firm size for productivities above z^{ext} but constant when measured relative to COGS. This is broadly in line with the empirical distribution of purchase obligations across the firm size distribution.

The intuition of this pattern is a follows. The size of a firm in the model is determined by its productivity. Large firms benefit enough from contracting purchase oblations to justify the fixed cost. As the financial constraints are defined relative to the ratio of profits and costs, firms that hedge choose the same hedging ratio PO/COGS. As more productive firms have larger costs, the dollar amount of PO is increasing in firm size.

[23]We use a discrete vector of productivities to obtain 20 firm sizes.

5.2.3 Exposure reduction

Using the calibrated model, we compute the implied commodity price elasticity. We compare this model statistic with the empirical estimates found above.

We compute the commodity price elasticity, conditional on PO use, as

$$\left.\frac{d\log\pi^m}{d\log q}\right|_{PO=0} = \int_q \frac{d\log\pi_i^m}{d\log q} dG(q) \frac{dH^m(z_i^m)}{\int_i dH^m(z_i^m)\mathbb{I}_{\bar{m}_i^m=0}}$$
$$\left.\frac{d\log\pi^m}{d\log q}\right|_{PO>0} = \int_q \frac{d\log\pi_i^m}{d\log q} dG(q) \frac{dH^m(z_i^m)}{\int_i dH^m(z_i^m)\mathbb{I}_{\bar{m}_i^m>0}}.$$

We also compute a counterfactual elasticity of PO firms if they did not use purchase obligations.

Table 5.3 shows model results for a 10% increase in commodity prices. The results are in line with the reduced-form estimates presented above. Also, we report the percentage of reduced exposure for firms using purchase obligations. This is computed as the ratio of the difference in commodity price elasticity for PO firms and the non-PO counterfactual elasticity divided by counterfactual elasticity.[24] This number is also in line with the estimates in Table 3.2.

Table 5.3: Model-implied commodity price elasticity

Non-PO Firms	PO Firms	PO Firms (counterfactual No PO)	Reduced Exposure (p.p.)	Reduced Exposure (%)
-0.1727	-0.1368	-0.1731	-0.0363	20.9748

Notes. The table reports the model-implied elasticity of profits to commodity prices. Coefficients are normalized to a 10% increase in commodity prices to compare with empirical results in Section 3.1.

The main takeaway of this section is that the model shows that firms can substantially reduce their exposure to commodity price risk by using purchase obligations.

5.2.4 Aggregate effects

In this section, we discuss the aggregate effects of hedging. We solve the model for two specifications. First, a benchmark model in which firms optimally set their PO, and second, a model in which firms are not allowed to use purchase obligations ($\kappa \to \infty$).

[24]Specifically, $\frac{-0.1731-(-0.1368)}{-0.1731} * 100 = 20.9748$

Table 5.4 reports the results. We show the percentage difference in the mean and standard deviation of aggregate variables between both models. The results show a positive effect on aggregate variables. Qualitatively, the results seem to suggest that risk-management policies have a positive impact on aggregate variables. For mean differences, both models yield approximately the same results, with a small positive difference for the PO model. Notice that the PO model yields a lower mean price level, but also a lower mean wage; however the results are not too large. On the other hand, when firms can trade purchase obligations, the standard deviation of aggregate variables is smaller. Only the price level is slightly more volatile. This suggests a substantial aggregate risk-exposure reduction.

Figures 5.3 shows the distribution of percentage differences across spot prices.[25] Firms can reduce interest expenses with the extra income from the forward operation. We see this in the profits figure. For low spot prices, firms reduce their income because there are financial losses associated with purchase obligations. For high spot prices, profits are larger in a world with purchase obligations precisely because of the additional financial income. Labor and output follow aggregate profits, because the representative consumer owns the firms. For prices, the response is smaller given that the interest rates change is smaller.

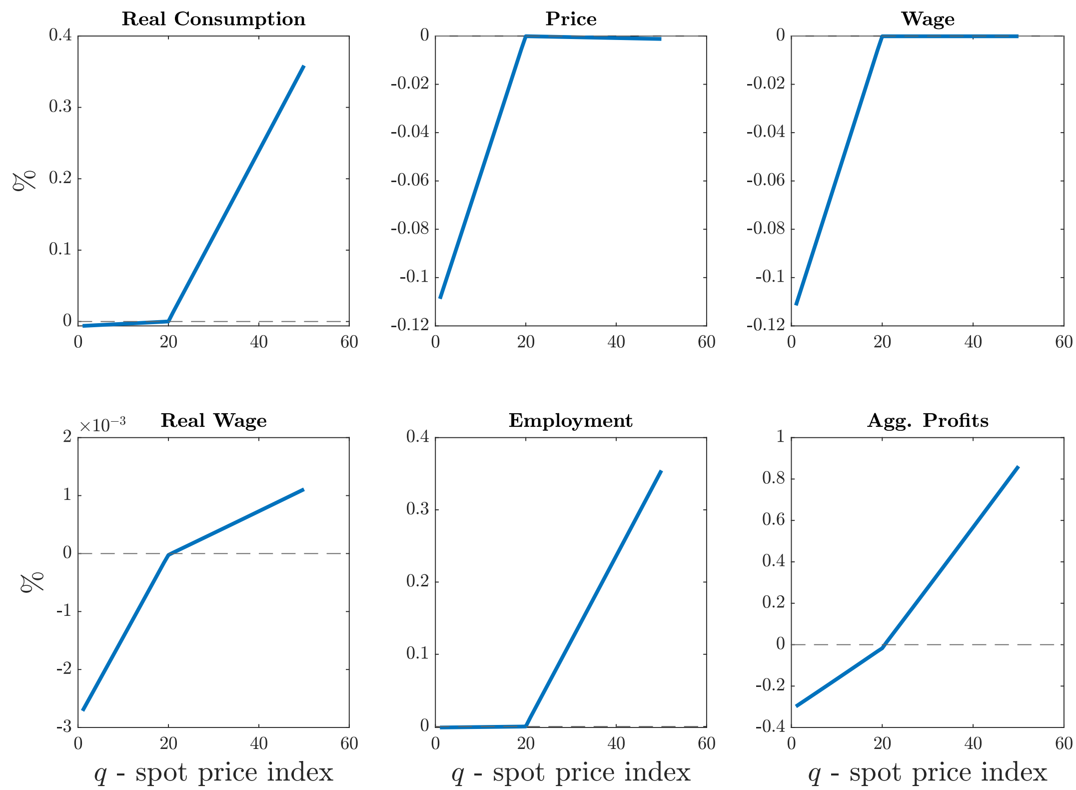
Table 5.4: Counterfactual Differences (%)

Variable	Mean	Std. Dev.
Real Consumption	0.024	-12.08
Employment	0.024	-28.326
Agg. Profits	0.012	-34.911
Price	-0.012	9.889
Wage	-0.012	-72.415
Real Wage	0	-0.287

Notes. The table shows differences in mean and standard deviation between the model with purchase obligations and the counterfactual in which firms are not allowed to trade these contracts. The second column computes the percentage difference between the std. dev. of the counterfactual and the benchmark.

[25] The simulations use a discrete vector of 50 spot prices.

Figure 5.3: Distribution of % differences



Notes. The figure shows the percentage difference between the calibrated model and the counterfactual in which firms are not allowed to trade purchase obligations for relevant aggregate variables.

5.2.5 Welfare implications

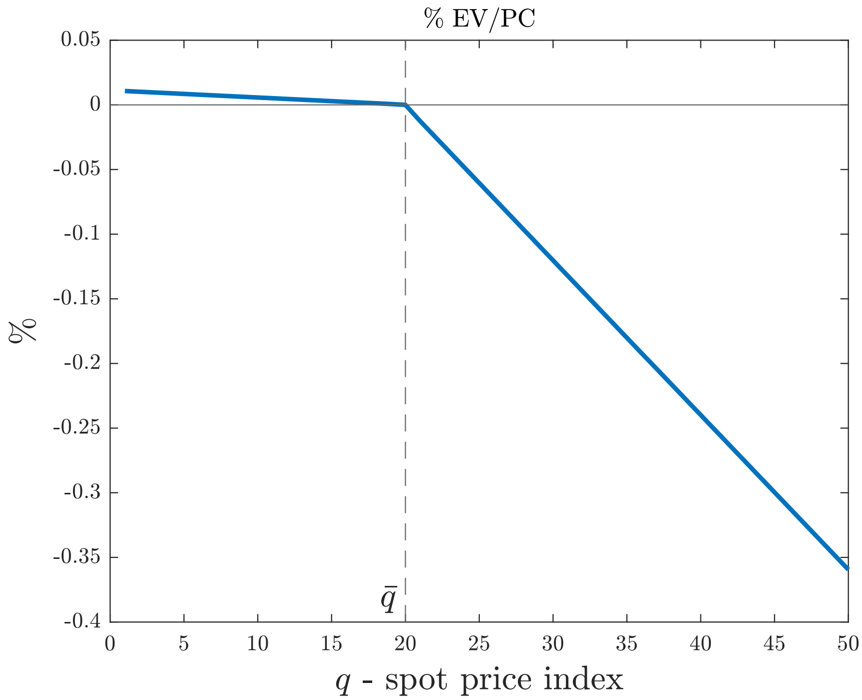
We compute the equivalent variation (EV) as a percentage of consumption the representative consumer is willing to sacrifice to allow firms to trade purchase obligations. The calibrated model predicts a 0.02 percentage points welfare gain in terms of mean consumption expenditures. This shows that the use of purchase obligations can improve welfare by partially offsetting the negative consequences of input price shocks.

We plot the distribution of EV as a share of consumption in Figure 5.4.[26] For low spot prices (below $\bar{q}$), there are financial losses associated with purchase obligations: hence the consumer is willing to receive extra income. For high spot prices (above $\bar{q}$), firms receive positive income from the forward and the consumer is better off. She values the welfare gains in this region and hence the EV is negative.

[26] For reference, we plot the weighted EV in Figure A.6 This includes the probability distribution of spot prices to measure the relative weight under the mean

Table 5.5 shows the relative magnitudes. The model predicts that the income from the region $q > \bar{q}$ is predominant. This implies that overall, the consumer is willing to sacrifice 0.024 percentage points of her consumption to allow firms to trade purchase obligations. These welfare changes are sizable. For comparison, these are about half of the welfare losses generated by the U.S. tariff implementation in 2018. For example, **?** find a 0.04% GDP loss.

Figure 5.4: Distribution of Equivalent Variation–No Weights



Notes. This figure shows the EV as a ratio of consumption for each spot price.

Table 5.5: Equivalent Variation–% of Consumption

Region	$q < \bar{q}$	$q > \bar{q}$	Total
EV	0.0012	-0.0248	-0.0236

5.2.6 Transmission of commodity price shocks

In this section we study the response of aggregate variables to commodity price shocks. We first show how aggregate variables react to commodity price shocks in Figure 5.5. Commodity price shocks negatively affect the economy by raising the cost of the material inputs needed for production. Also, since face firms face financial constraints, these shocks increase the share of financially constrained firms. This generates adverse conditions for firms and, in turn, enlarges the consequences of commodity price shocks. For instance, a 10% commodity price shock reduces employment and real GDP by 0.1% and 0.28%, respectively. These are in line with estimates for oil shocks in employment (e.g. ?) and the empirical estimates found in the beginning of this paper.

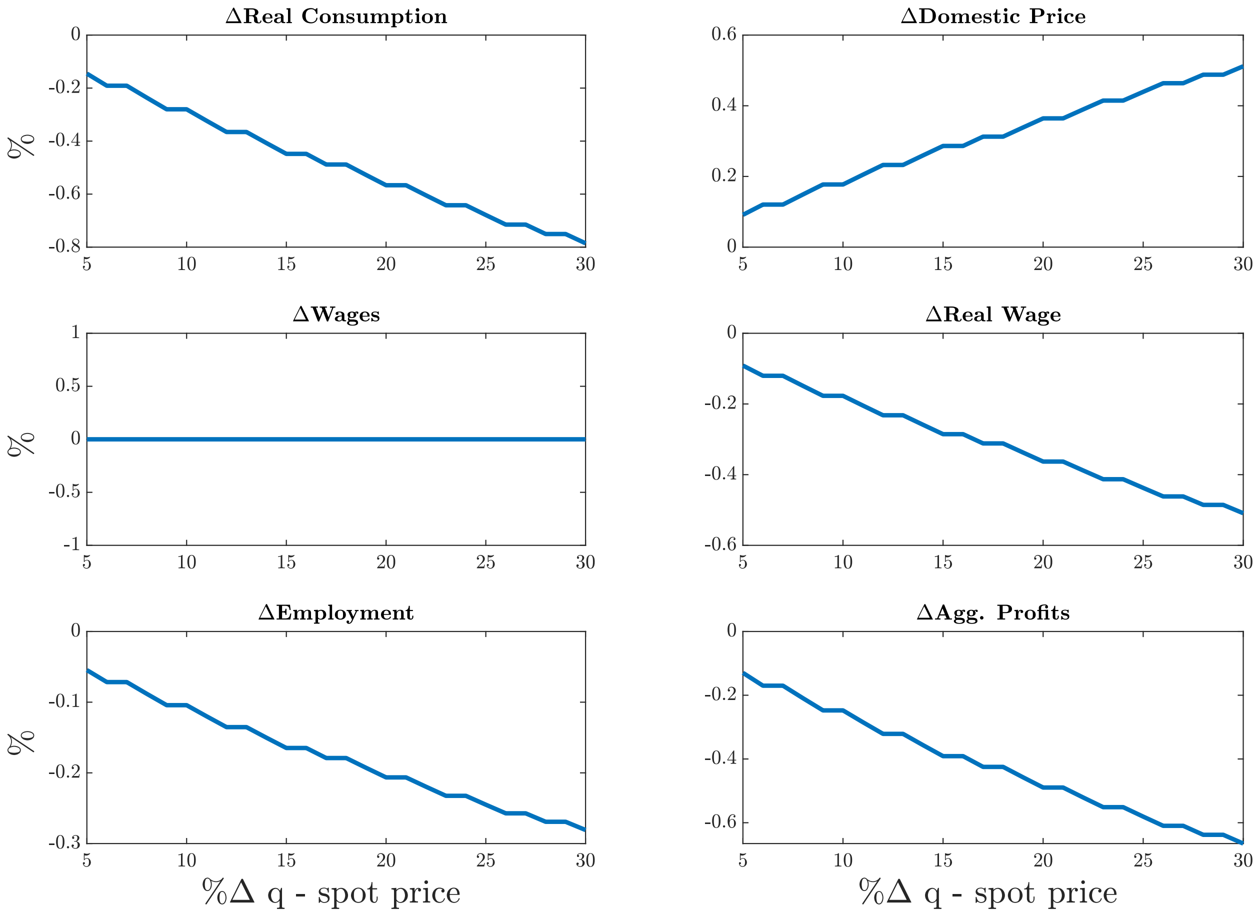
Next, we compare the transmission in the calibrated model with purchase obligations with the counterfactual in which firms are not allowed to engage in these contracts. We compute the percentage change in each aggregate variable for difference percentage changes in spot prices. We repeat this calculation for the two model and plot the relative percentage change in Figure 5.6. We also report the p.p. difference in Figure A.7. For illustration, Table 5.6 shows the results for a 10% increase in commodity prices.

Table 5.6: Agg. Elasticity

Agg. Variable	%Δ (No PO)	%Δ (PO)	p.p diff.	% diff.
Real Consumption	-0.3518	-0.2799	0.0718	25.636
Employment	-0.1753	-0.1042	0.0710	68.189
RGDP	-0.3528	-0.2812	0.0716	25.50

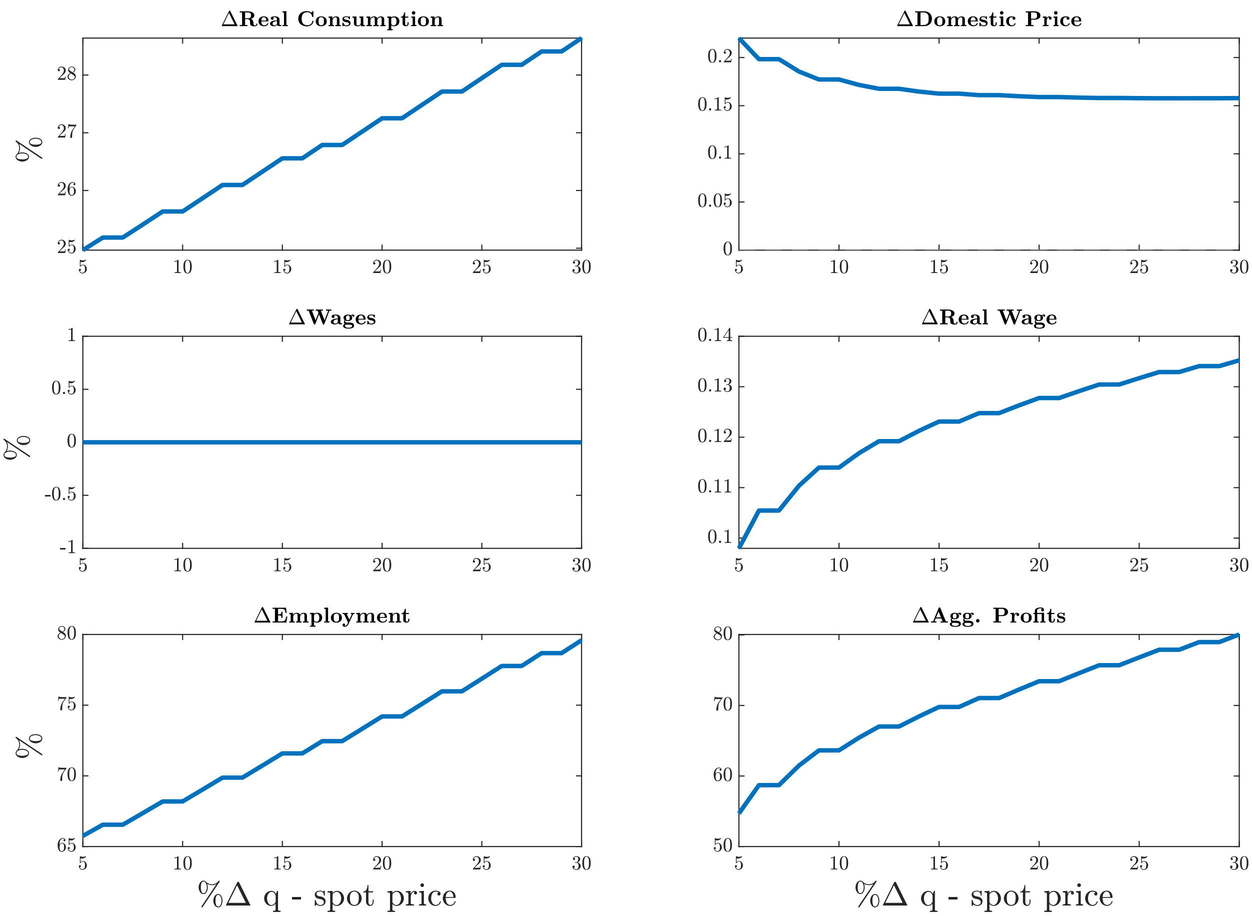
Firms are less sensitive to changes in commodity prices when they can trade purchase obligations, as we have seen in previous sections. This smaller response at the firm level is translated into the aggregate economy due to extra income from the forward operation. Quantitatively, we find a positive aggregate contribution of purchase obligations to dampening the transmission of commodity price shocks. For instance, aggregate real consumption and real GDP react 25.6% less in the model with purchase obligations when the spot price increases by 10%.

Figure 5.5: Transmission–Commodity Price Shock



Notes. This figure shows the transmission of commodity price shocks when firms can trade purchase obligations. The x-axis shows the percentage change in spot price. The y-axis shows the percentage change of each aggregate variable compared with the benchmark economy (calibrated to match stylized facts for the U.S. in 2012)

Figure 5.6: Relative Transmission–Commodity Price Shock, % difference



Notes. This figure shows the relative transmission of commodity price shocks. The x-axis shows the percentage change in spot prices. The y-axis shows the percentage difference between the response of each aggregate variable to a change in the spot price in the two models (PO vs non-PO).

5.2.7 Unpacking the mechanism

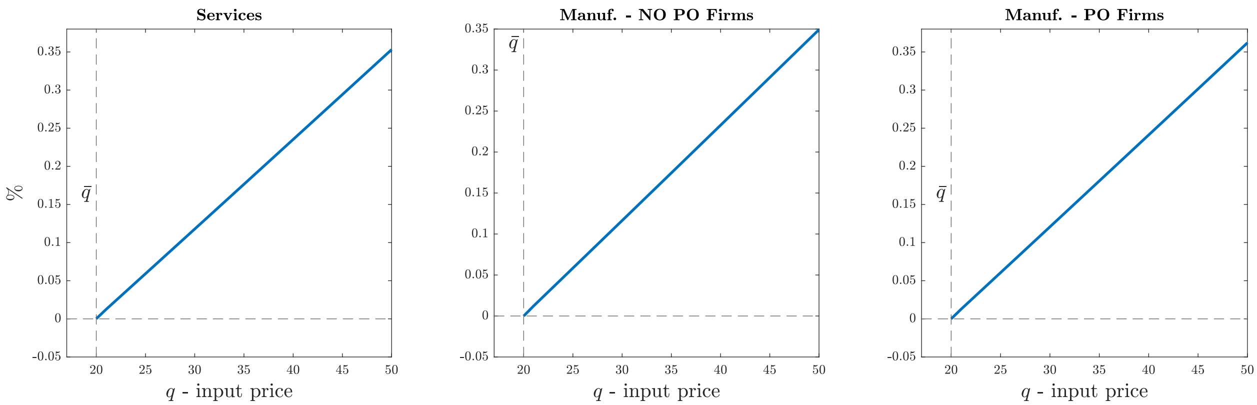
In this section we explore quantitative the mechanism behind why purchase obligations can dampen the transmission of commodity price shocks. First, there are strong reallocation effects across firms. Firms in the service sector benefit because the income effect from purchase obligations increases demand of all goods. This logic also applies to the manufacturing sector. However, relative prices between PO and non-PO firms change implying that only PO firms reap the aggregate benefits.

Figure 5.7 shows the employment difference between the calibrated model and the counterfactual without purchase obligations for each spot price. Each subplot shows difference subsets of firms: services, manufacturing engaging in PO and manufacturing non-PO. For clarity, we divided the plot in two panels: (a) for below mean spot prices ($q < \bar{q}$) and (b) for above mean ($q > \bar{q}$). Several facts are worth mentioning. First, firms in the services sector benefit from purchase obligations when the income effect is positive i.e. $q > \bar{q}$. Second, the employment change in firms in the manufacturing sector that do not hedge is always positive. When spot prices are below mean, PO manufacturing firms increase their prices to adjust to the PO losses and non-PO firms have a competitive advantage. When spot prices are above mean, the extra PO income has spillover effects on non-PO firms as overall demand increases relative to the counterfactual. Third, PO firms are the ones that benefit the most from trading purchase obligations. These firms are between 0.05 to 0.35 percent larger when $q > \bar{q}$ compared to the counterfactual. For situations where the forward operation delivers negative income ($q < \bar{q}$), these firms become smaller.

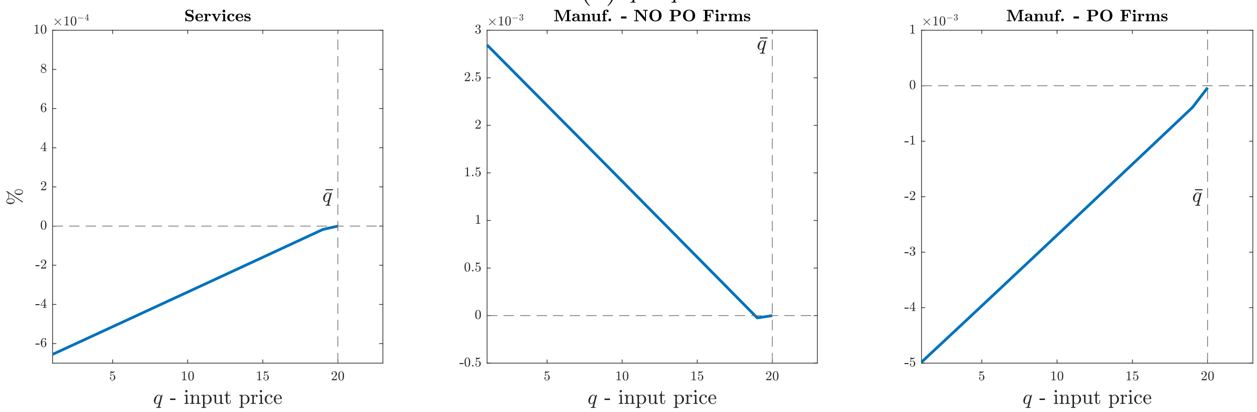
In second place, the transmission results from the previous sections can be decomposed into two channels: (i) a direct effect coming from the income change from purchase obligations and (ii) general equilibrium effects from this additional income. In Figure 5.9 we plot the decomposition for different commodity price shocks.

In a nutshell, when firms trade purchase obligations they receive additional financial income from the forward operation. This extra income is consumed by households. The direct effect only accounts for the transmission differential in consumption only taking this effect into account. On the other hand, the consumer will increase expenditures from all firms. This in fact increases production and income further more. In general equilibrium, the existence of this income effect generates additional effects. For instance, after a 10% in commodity prices, our results show that 1% of the transmission difference is due to the income effect. The general equilibrium effects account for the remaining share.

Figure 5.7: Employment changes



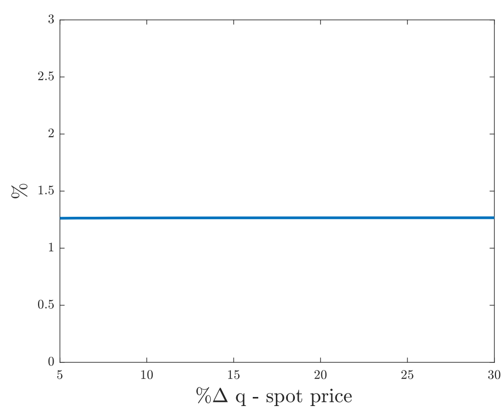
(a) $q < \bar{q}$



(b) $q > \bar{q}$

Notes. This figure shows the percentage difference in employment across firm groups between the calibrated economy and the counterfactual without purchase obligations. For each firm group we compute the average within firms. Panel (a) shows the change for low spot prices ($q < \bar{q}$) and Panel (b) for high spot prices ($q > \bar{q}$)

Figure 5.9: Transmission Decomposition



Notes. This figure shows the decomposition of the transmission difference between the model with PO and the counterfactual without PO. For each change in commodity price shocks studied in Fig: A.7 we split the effects between the direct effect and the general equilibrium spillover.

6 Conclusion

In this paper we study the aggregate implications of risk-management policies. In particular, we leverage a novel dataset on supply contracts with fixed prices for public companies in the manufacturing sector in the United States. Firms rely on these tools to reduce their exposure to input price risk. We find a substantial decrease in exposure to input price changes for firms using these contracts. Also, we develop a general equilibrium model that features purchase obligations to show how firm risk-management policies can insure the economy against input price shocks and increase welfare.

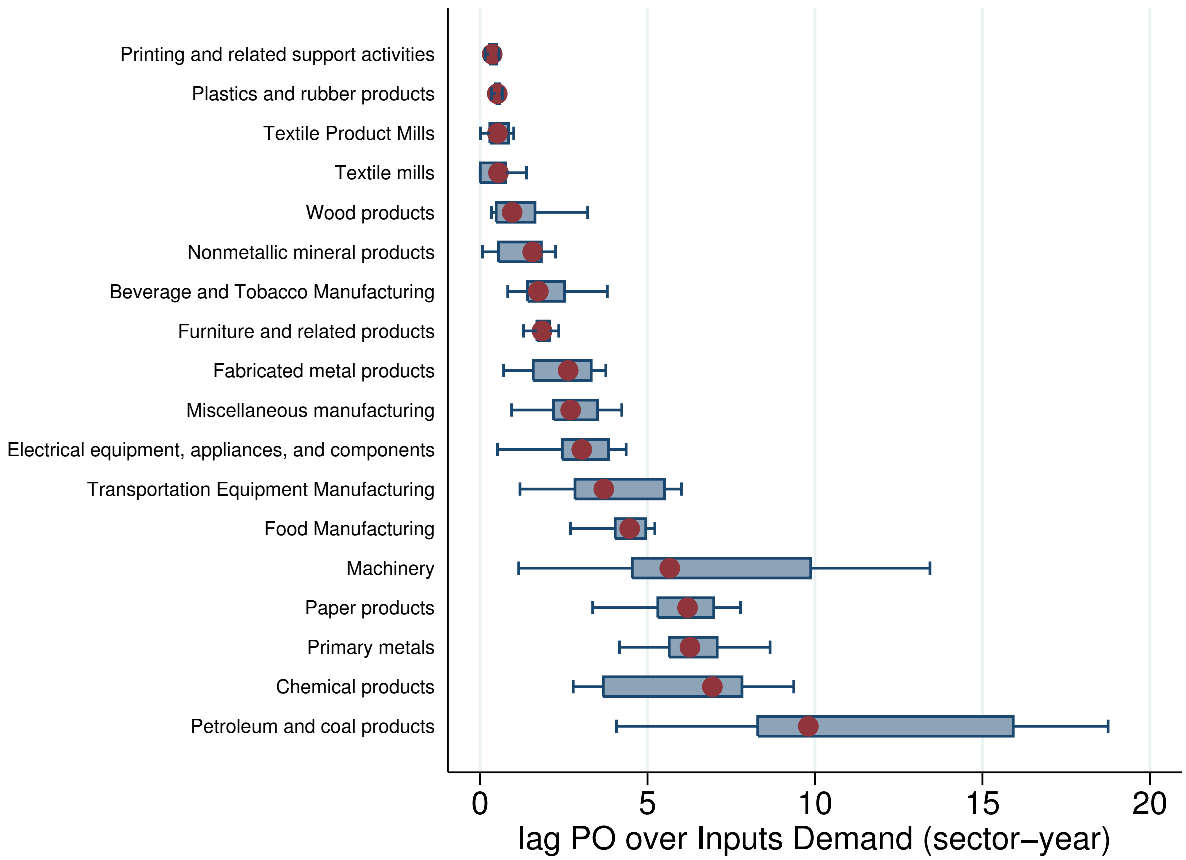
Our results suggest that governments and central banks should pay close attention to the risk-management strategies used by corporations. These provide strong exposure reduction at the firm level, which can be transmitted to the aggregate economy. Consumers can reap the benefits of these policies by facing a lower volatility of consumption and employment.

Moreover, the results have suggest several policy implications. Central Banks have been actively managing interest rates to control inflation due to surges in commodity prices. This study suggests that although short-run interest rates could control inflation, Central Banks should take risk-management policies into account because the recession induced could be lower. Hence, the trade-off between controlling inflation and economic activity could be improved, since the financial market can help overcome the negative consequences of these shocks. On the other hand, our results also suggest that these instruments could be under-supplied in equilibrium. For instance,only a small measure of firms in the model and the data engage in purchase obligations, although the benefits seem to be large. This suggests that there might be room for policy interventions to reduce participation costs: for example, regulatory costs.

References

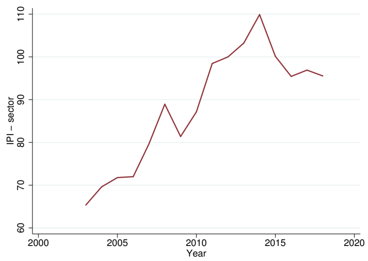
A Additional Tables and Figures

Figure A.1: PO–Sector Intensity

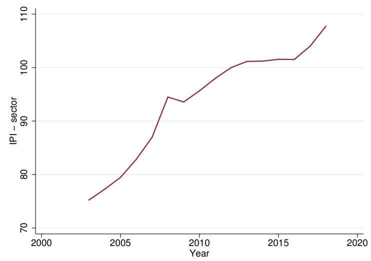


Notes. The figure shows the distribution over time of the share (%) of total sector purchase obligations (lag) over total sector material inputs demand. Red dots represent the median across time.

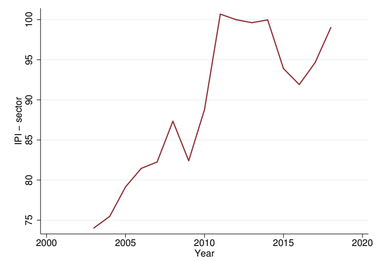
Figure A.2: Input Price Index by Sector



(a) Food Manufacturing

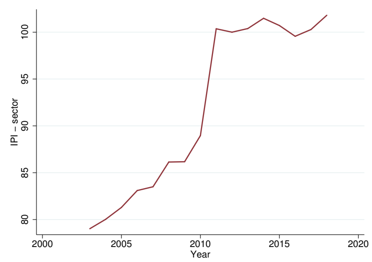


(b) Beverage and Tobacco Manufacturing

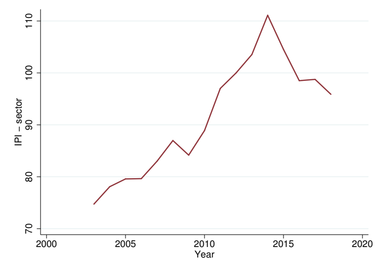


(c) Textile mills

(d) Textile Product Mills

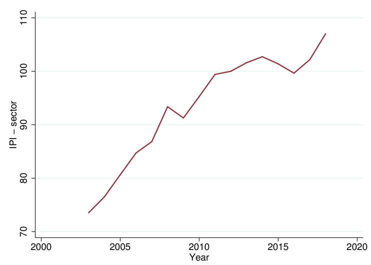


(e) Apparel Manufacturing

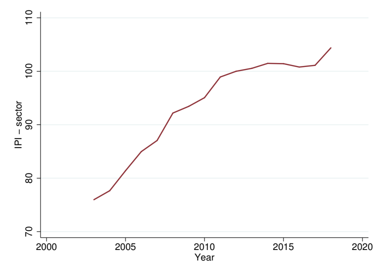


(f) Leather and Allied Product Manufacturing

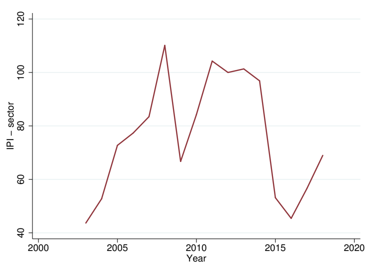
(g) Wood products



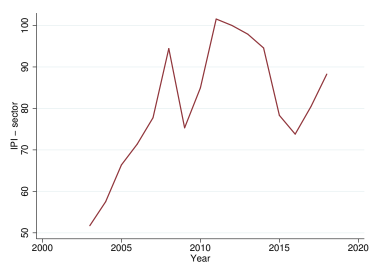
(h) Paper products



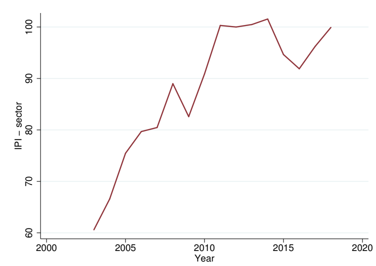
(i) Printing and related support activities



(j) Petroleum and coal products

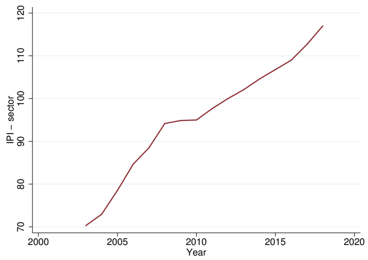


(k) Chemical products

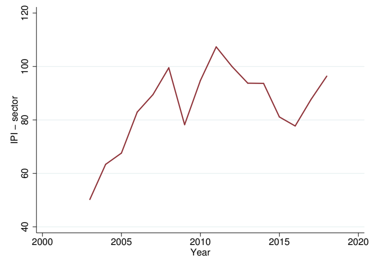


(l) Plastics and rubber products

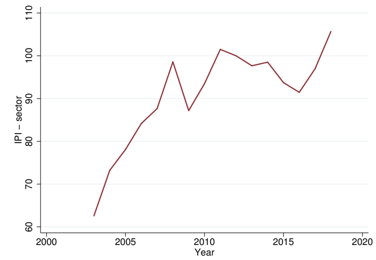
Notes. This figure shows the evolution of the input price index. It is constructed using material shares from the Economic Census 2012 for manufacturing sectors and commodity price indexes from the BLS and World Bank. Base year 2012.



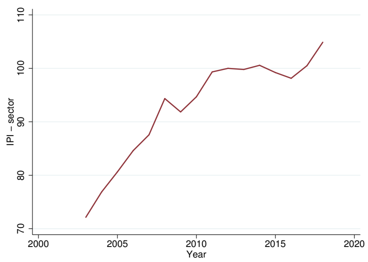
(m) Nonmetallic mineral products



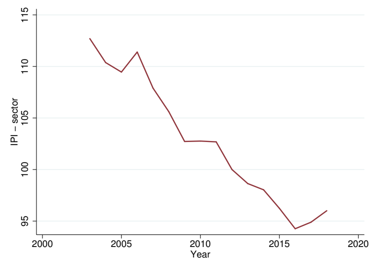
(n) Primary metals



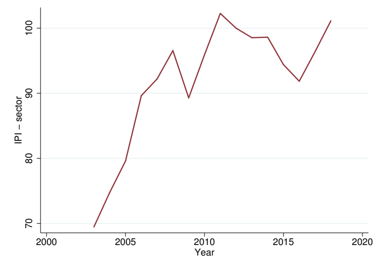
(o) Fabricated metal products



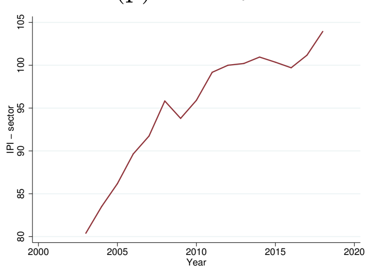
(p) Machinery



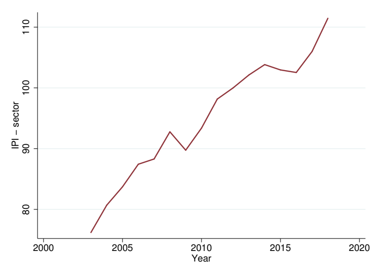
(q) Computer and electronic products



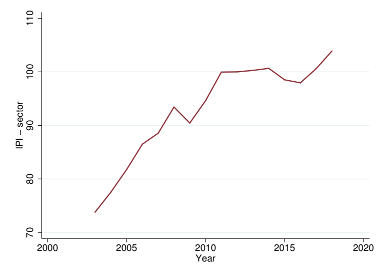
(r) Electrical equipment, appliances, and components



(s) Transportation Equipment Manufacturing



(t) Furniture and related products



(u) Miscellaneous manufacturing

Table A.1: IPI–Material Shares

Sector (NAICS-3)	Sector Name	Share Material Used (%)	Material Description
311	Food Manufacturing	9.31	Cattle slaughtered
312	Beverage and Tobacco Manufacturing	15.7	Other concentrated liquid beverage bases
313	Textile mills	11.8	Raw cotton fibers
314	Textile Product Mills	19.44	Nylon filament yarn
315	Apparel Manufacturing	37.63	Broadwoven fabrics
316	Leather and Allied Product Manufacturing	47.35	Hides, skins, and pelts
321	Wood products	19.47	Softwood logs and bolts
322	Paper products	47.78	Paper and paperboard
323	Printing and related support activities	17.76	Coated paper
324	Petroleum and coal products	47.58	Foreign crude petroleum
325	Chemical products	10.18	Agricultural products
326	Plastics and rubber products	45.67	Plastics resins
327	Nonmetallic mineral products	16.89	Portland and blended cements
331	Primary metals	14.05	All other steel shapes and forms
332	Fabricated metal products	7.45	Steel sheet and strip
333	Machinery	7.97	Iron and steel castings
334	Computer and electronic products	12.87	Semiconductors
335	Electrical equipment, appliances, and components	6.51	Electronic-type components
336	Transportation Equipment Manufacturing	8.61	Gasoline engines and parts
337	Furniture and related products	6.9	Other woven upholstery fabrics
339	Miscellaneous manufacturing	19.79	Surgical and orthopedic supplies

Notes. This table shows the share of total materials costs of the most important commodity purchased in each sector. Numbers were computed using the Economic Census 2012.

Table A.2: Linear Prob. Model Ind. PO

	(1) Ind PO	(2) Ind PO
Working Capital/Assets	0.0161^{+}	0.0318
	(0.00932)	(0.0240)
Retained earnings/Assets	-0.000951	0.00256***
	(0.000839)	(0.000610)
EBIT/Assets	-0.0146	-0.0350***
	(0.0165)	(0.00819)
Market value/Liabilities	-0.0000536	-0.000354
	(0.000718)	(0.000380)
Sale/Assets	0.0137	0.0256
	(0.0195)	(0.0160)
log Cash	0.0200**	0.00540
	(0.00754)	(0.00475)
log Assets	0.0650***	0.0547***
	(0.0124)	(0.0138)
log Inv Raw Mat	-0.00930	0.00667
	(0.00915)	(0.00728)
Constant	0.165**	0.229**
	(0.0582)	(0.0840)
Observations	12654	12475
R^2	0.093	0.786
FE	No	Firm

Notes. This table reports the estimation of a linear probability model of the PO indicator using firm characteristics and fixed effects as controls. The estimated equation is $\mathbb{I}_{PO_{i,t}>0} = \alpha + \sum_m \beta_m \, Control_{m,i,t} + error_{i,t}$. Standard errors are clustered at the firm level. $^{+}$ $p < 0.10$, * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table A.3: Input Price Elasticity Estimation–Other Measures of Firm Value

	(1) Change NI/AT	(2) Change EBIT/AT	(3) Change EBITDA/AT
Change Sector IPI	-0.00113*	-0.0000645	-0.000156
	(0.000515)	(0.000399)	(0.000407)
Change Sector IPI $\times$ lag Ind PO	0.00166**	0.000940*	0.000955*
	(0.000537)	(0.000421)	(0.000426)
Constant	-0.00227*	-0.00268***	-0.00252***
	(0.000889)	(0.000620)	(0.000625)
Observations	13771	13760	13664
R^2	0.001	0.003	0.003

Notes. This table reports the estimation of input price elasticity using additional measures of firm value. The estimated equation is 2. The Firm value measures are Net Income, Earnings Before Interest, and Taxes and Earnings Before Interest, Taxes, and Depreciation. All variables are normalized by total assets. Regression drops top and bottom 1% outliers for Δ Firms Value / Assets. Standard errors are clustered at the firm level and included in parenthesis. $^{+}$ $p < 0.10$, * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table A.4: Input-price elasticity estimation–other measures of firm value–FE

	(1) Change NI/AT	(2) Change EBIT/AT	(3) Change EBITDA/AT
Change Sector IPI	-0.00133*	-0.000159	-0.000270
	(0.000552)	(0.000466)	(0.000478)
Change Sector IPI $\times$ lag Ind PO	0.00168**	0.000972*	0.00102*
	(0.000583)	(0.000494)	(0.000503)
Constant	-0.00190***	-0.00248***	-0.00231***
	(0.000304)	(0.000240)	(0.000237)
Observations	13581	13570	13472
R^2	0.093	0.110	0.112
FE	Firm	Firm	Firm

Notes. This table reports the estimation of input-price elasticity using additional measures of firm value. The estimated equation is 2. The firm value measures are: Net Income, Earnings before interest and taxes and Earnings before interest taxes and depreciation. All variables are normalized by total assets. Additional controls include fixed effects. Regression drops top and bottom 1% outliers for Δ Firms Value / Assets. Standard errors are clustered at the firm level and included in parenthesis. $^{+}$ $p < 0.10$, * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table A.5: Input-price elasticity estimation–PO/COGS, Returns

	(1) log Returns	(2) log Returns	(3) log Returns
Change Sector IPI	-0.0111***	-0.0122***	-0.0128***
	(0.00109)	(0.00114)	(0.00122)
Change Sector IPI × lag PO/COGS	0.0169*	0.0169*	0.0153*
	(0.00681)	(0.00697)	(0.00766)
Constant	0.0175***	0.0188***	0.0213***
	(0.00465)	(0.00459)	(0.00115)
Observations	12989	12989	12826
R^2	0.014	0.019	0.155
FE	None	NAICS 3	Firm

Notes. This table reports the estimation of inputprice elasticity allowing for different coefficients according to the hedging intensity. The estimated equation is 3. The firm value measure used in this regression is stock returns. Each column includes different fixed effects. Regression drops top 2% outliers for PO/COGS. Standard errors are clustered at the firm level and included in parenthesis. $^{+}$ $p < 0.10$, * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table A.6: Input-price elasticity estimation–PO/COGS, NI

	(1) Change NI/AT	(2) Change NI/AT	(3) Change NI/AT
Change Sector IPI	-0.000337	-0.000363	-0.000503*
	(0.000229)	(0.000234)	(0.000250)
Change Sector IPI × lag PO/COGS	0.00375*	0.00389*	0.00348*
	(0.00156)	(0.00156)	(0.00170)
Constant	-0.00212+	-0.00210+	-0.00172
	(0.00123)	(0.00123)	(0.00126)
Observations	13541	13541	13352
R^2	0.000	0.001	0.097
FE	None	NAICS 3	Firm

Notes. This table reports the estimation of input price elasticity allowing for different coefficients according to the hedging intensity. The estimated equation is 3. The firm value measure used in this regression is net income. Each column includes different fixed effects. Regression drops top and bottom 1% outliers for Δ Firms Value / Assets and top 2% outliers for PO/COGS. Standard errors are clustered at the firm level and included in parenthesis. $^{+}$ $p < 0.10$, * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table A.7: Input Price Elasticity Estimation–PO/COGS, EBIT

	(1) Change EBIT/AT	(2) Change EBIT/AT	(3) Change EBIT/AT
Change Sector IPI	0.000458**	0.000453**	0.000313*
	(0.000143)	(0.000146)	(0.000155)
Change Sector IPI × lag PO/COGS	0.00181+	0.00189+	0.00241*
	(0.000966)	(0.000968)	(0.00105)
Constant	-0.00260***	-0.00260***	-0.00229**
	(0.000764)	(0.000765)	(0.000773)
Observations	13530	13530	13340
R^2	0.002	0.003	0.114
FE	None	NAICS 3	Firm

Notes. This table reports the estimation of input price elasticity allowing for different coefficients according to the hedging intensity. The estimated equation is 3. The firm value measure used in this regression is EBIT. Each column includes different fixed effects. Regression drops top and bottom 1% outliers for Δ Firms Value / Assets and top 2% outliers for PO/COGS. Standard errors are clustered at the firm level and included in parenthesis. $^{+}$ $p < 0.10$, * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table A.8: Input Price Elasticity Estimation–PO/COGS, EBITDA

	(1) Change EBITDA/AT	(2) Change EBITDA/AT	(3) Change EBITDA/AT
Change Sector IPI	0.000369**	0.000361*	0.000214
	(0.000140)	(0.000143)	(0.000152)
Change Sector IPI × lag PO/COGS	0.00194*	0.00204*	0.00272**
	(0.000948)	(0.000950)	(0.00103)
Constant	-0.00237**	-0.00237**	-0.00206**
	(0.000751)	(0.000752)	(0.000758)
Observations	13434	13434	13242
R^2	0.002	0.003	0.116
FE	None	NAICS 3	Firm

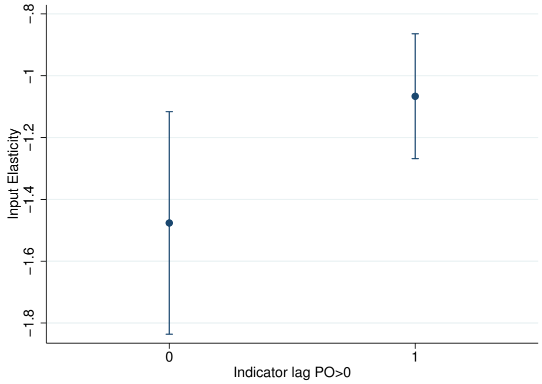
Notes. This table reports the the estimation of input-price elasticity allowing for different coefficients according to the hedging intensity. The estimated equation is 3. The firm value measure used in this regression is EBITDA. Each column includes different fixed effects. Regression drops top and bottom 1% outliers for Δ Firms Value / Assets and top 2% outliers for PO/COGS. Standard errors are clustered at the firm level and included in parenthesis. $^{+}$ $p < 0.10$, * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table A.9: Input Price Elasticity Estimation–Ups and Downs, Returns

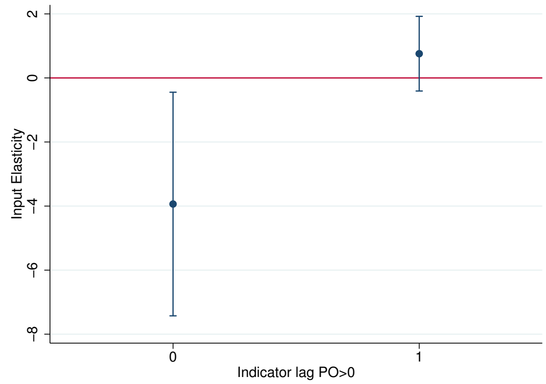
	(1) log Returns	(2) log Returns
Change Sector IPI	0.00105	0.00930***
	(0.00252)	(0.00257)
Change Sector IPI × lag Ind PO	-0.00301	-0.00343
	(0.00269)	(0.00277)
Change Sector IPI (+)	-0.0266***	-0.0469***
	(0.00397)	(0.00425)
Change Sector IPI (+) × lag Ind PO	0.0128**	0.0131**
	(0.00395)	(0.00412)
Constant	0.0524***	0.0985***
	(0.00615)	(0.00690)
Observations	13237	13237
R^2	0.023	0.038
FE	None	NAICS 3

Notes. This table reports the estimation of input price elasticity allowing for different coefficients for increases or decreases of input prices. The estimated equation is 4. Standard errors are clustered at the firm level and included in parenthesis. $^{+}$ $p < 0.10$, * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

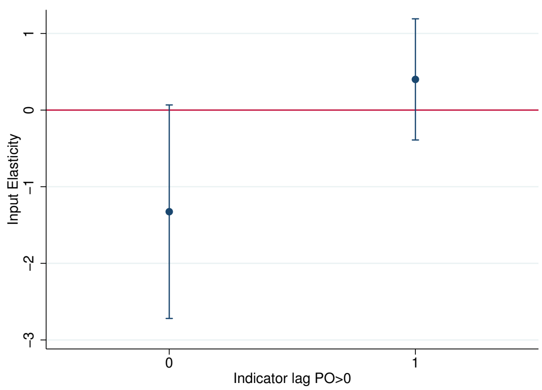
Figure A.3: Input Price Elasticity Estimation–Indicator PO



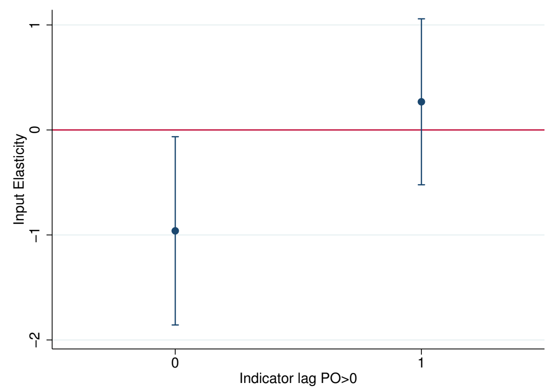
(a) Returns



(b) Net Income



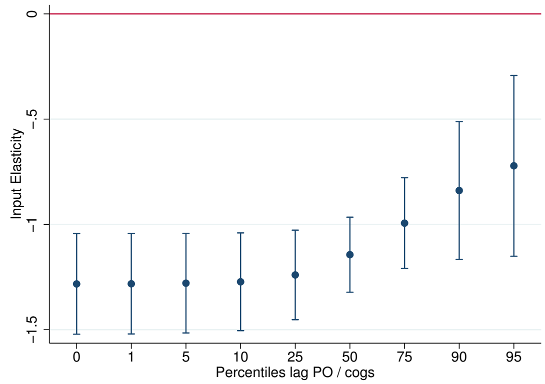
(c) EBIT



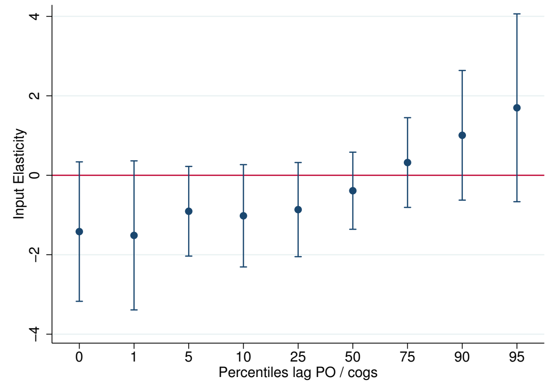
(d) EBITDA

Notes. This figure reports estimates of the input price elasticity for several firm value measures between PO and non-PO firms. Coefficients are taken from Table 3.1 for returns and Table A.3 for NI, EBIT, and EBITDA (Column 3). Standard errors were computed using the delta method. For NI, EBIT and EBITDA we normalize the coefficients by the median firm value measure over total assets for better interpretation.

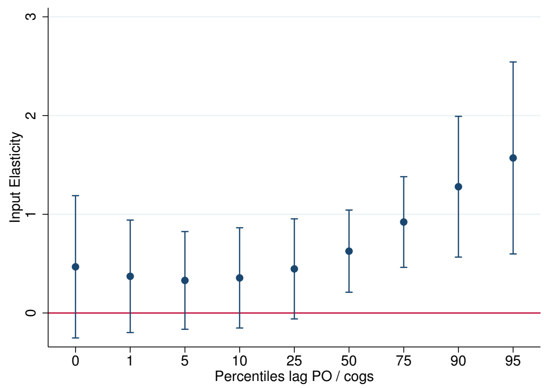
Figure A.4: Input Price Elasticity Estimation–PO/COGS



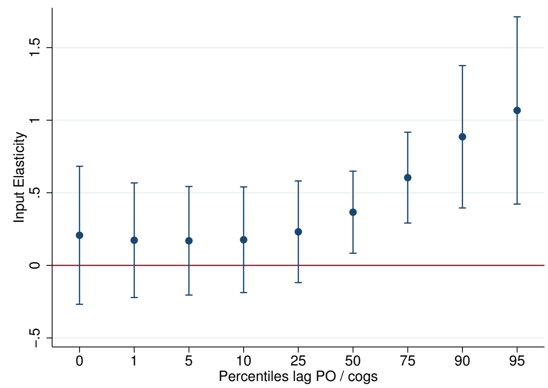
(a) Returns



(b) Net Income



(c) EBIT



(d) EBITDA

Notes. This figure reports estimates of the input elasticity for several firm value measures for different values of PO/COGS. Coefficients are taken from Tables A.5 (Returns), A.6 (NI), A.7 (EBIT), and A.8 (EBITDA). All coefficients come from Column 3. Standard errors were computed using the delta method. For NI, EBIT, and EBITDA, we normalize the coefficients by the median firm value measure over total assets for better interpretation.

Table A.10: Summary Statistics Financial Characteristics

Variable	median	lowest 10%	lowest 25%	top 75%	top 90%
log Assets	6.407	3.797	5.013	7.811	9.011
Working Capital / Total Assets	0.284	0.059	0.153	0.466	0.632
Retained Earnings/ Total Assets	0.131	-1.870	-0.309	0.395	0.609
EBIT / Total Assets	0.076	-0.133	0.015	0.126	0.187
Sales / Total Assets	0.918	0.397	0.630	1.306	1.739
Market Value of Equity / Book value of Liabilities	2.638	0.608	1.253	6.205	14.939

Notes. This table shows summary statistics for the covariates used as controls in regression 5.

Table A.11: Commodity price elasticity

Panel A: bottom 25% PO/COGS (2.8%)

	median	lowest 10%	lowest 25%	top 75%	top 90%
No PO firm elasticity	-1.28	-1.85	-1.62	-0.87	-0.45
PO firm elasticity	-1.24	-1.82	-1.58	-0.83	-0.42
p.p. difference	0.037	0.037	0.037	0.037	0.037
% difference	2.88%	2.00%	2.28%	4.26%	8.16%

Panel B: median PO/COGS (9%)

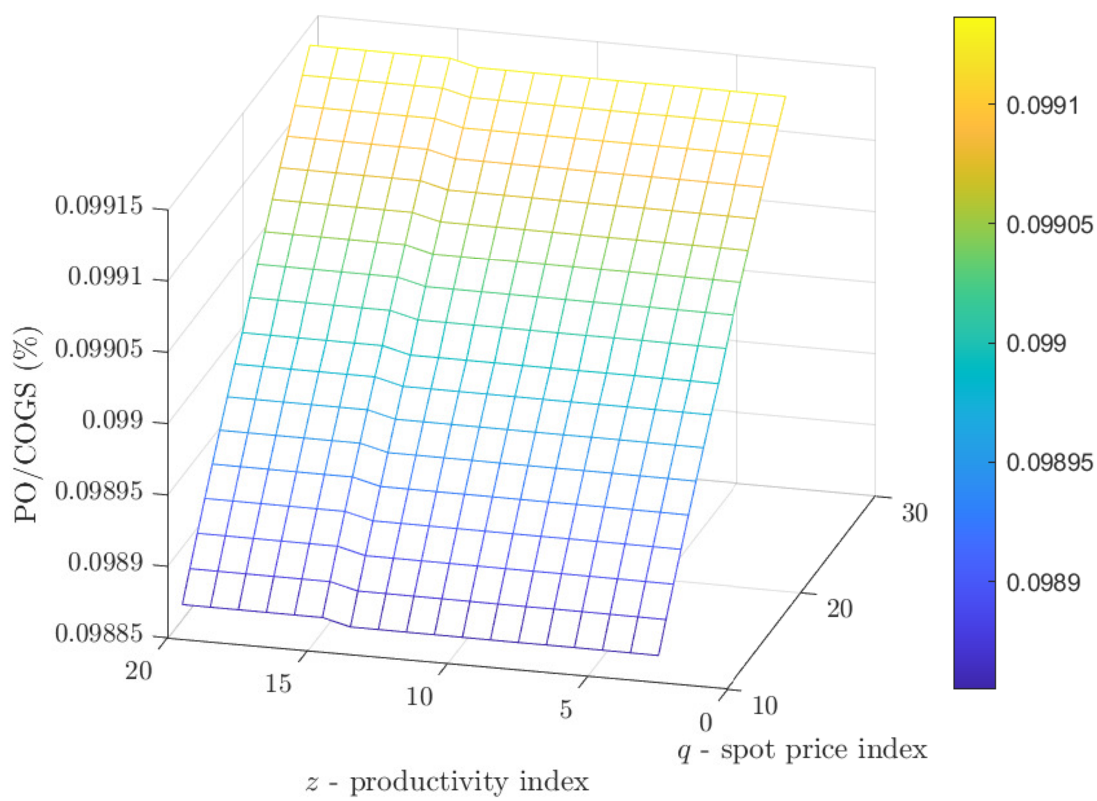
	median	lowest 10%	lowest 25%	top 75%	top 90%
No PO firm elasticity	-1.28	-1.85	-1.62	-0.87	-0.45
PO firm elasticity	-1.16	-1.73	-1.50	-0.75	-0.33
p.p. difference	0.119	0.119	0.119	0.119	0.119
% difference	9.27%	6.41%	7.33%	13.69%	26.24%

Panel C: top 75% PO/COGS (19%)

	median	lowest 10%	lowest 25%	top 75%	top 90%
No PO firm elasticity	-1.3	-1.9	-1.6	-0.9	-0.5
PO firm elasticity	-1.0	-1.6	-1.4	-0.6	-0.2
p.p. difference	0.251	0.251	0.251	0.251	0.251
% difference	19.58%	13.54%	15.48%	28.90%	55.39%

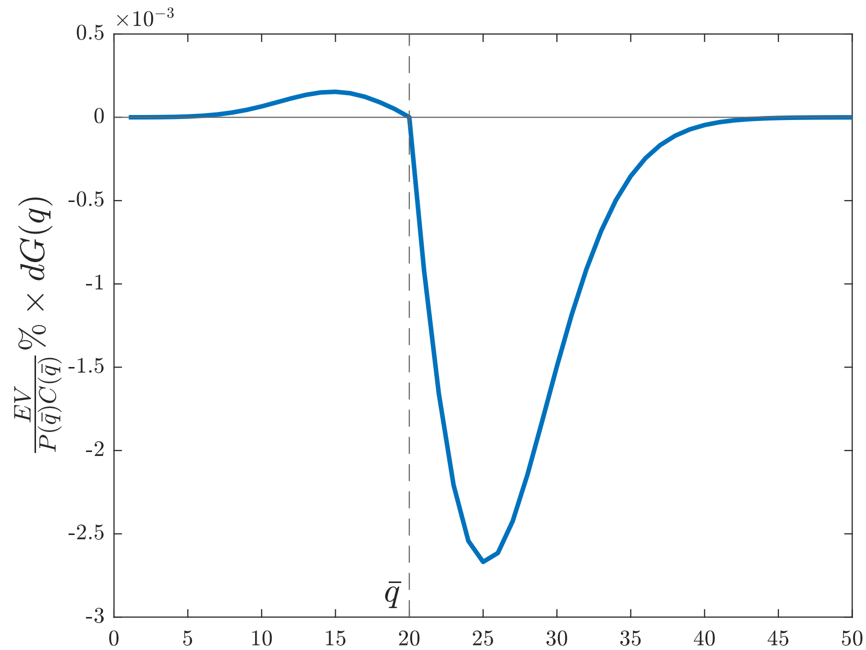
Notes. The table shows the implied commodity price elasticity using coefficients from Table 3.3 (column 3) and covariates from Table A.10. Each panel computes the implied commodity price elasticity for different covariates values across the firm-size distribution (see Table A.10). For the PO elasticity we use different values of the PO/COGS distribution: bottom 25% (2.8%, Panel A); median (9%, Panel B); top 75% (19%, Panel C). The % diff. elasticity PO is computed as the % difference between No PO and PO elasticities for each column within panel.

Figure A.5: Model-implied PO/COGS



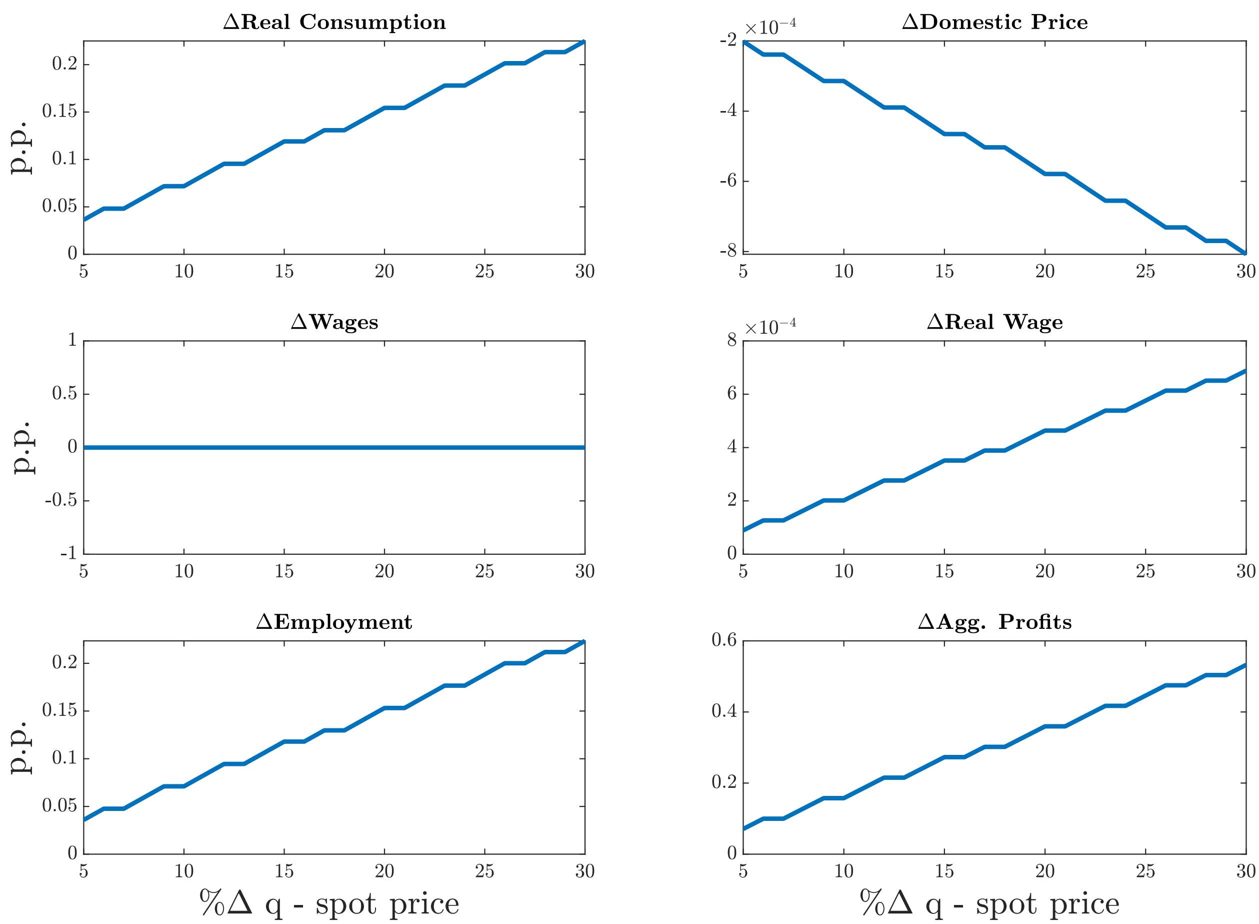
Notes. This figure reports the ratio of purchase obligations value to total cost in the quantitative exercise for different productivities and spot prices.

Figure A.6: Distribution of Equivalent Variation



Notes. This figure shows the EV as a ratio of consumption weighted by the density of spot prices.

Figure A.7: Relative Transmission–Commodity Price Shock, p.p. difference



Notes. This figure reports the p.p. difference in transmission between the model with purchase obligation and the counterfactual in which firms are not allowed to trade these contracts.

B Data Collection

The dataset used in this paper is a combination of firm balance sheet, industry, and text-based characteristics. We constructed the dataset in several steps:

1. **Scope.** Using COMPUSTAT, we downloaded the CIK (SEC identifier) for all public firms in the manufacturing sector (NAICS 31-33). The Securities and Exchange Commission keeps an online repository of all filings starting in 1993. These can be accessed through the website `https://www.sec.gov/Archives/edgar/full-index/`. We downloaded the header of all reports filed between 2003 and 2019 for companies with CIKs found on COMPUSTAT and belonging to the manufacturing sector. We kept only company-year observations with a 10-K report in the EDGAR repository.

2. **Firm Characteristics.** We used COMPUSTAT to obtain earnings and costs measures used throughout the paper and CRSP for stock returns.

3. **Purchase Obligations.** We constructed the purchase obligations dataset in three steps, following **?** and **?**.

   For each company-year in the scope, we downloaded the purchase obligation table using a scrapping algorithm in Python. For each 10-K in **Scope**, the algorithm reads through the 10-K and finds the table with the purchase obligation amount. The keywords used were `Purchase Obligations`, `Purchase Commitments`, `Purchase Orders`, and `Contractual Obligations`.[27]

   Companies do not follow a strict reporting procedure and therefore some adjustments are needed. In the first place, the unit of account of PO is problematic. Some companies report explicitly the unit (dollars, thousands or millions), but others fail to do so. We solve this issue by extracting the unit of account from the table if it is available. If the unit is missing, we compute the ratio PO/Cost of Goods Sold and define the unit of measure based on three thresholds.

Unit	Threshold $PO/COGS$
Millions	< 0.45
Thousands	< 2.7 & > 0.45
Dollars	$> 10,200$

   We verify that this process correctly accounts for the unit of measure by manually checking the annual reports of about 1% of the sample.

4. **Input Price Index.** We constructed a Laspeyres price index from materials used by sector using the Economics Census 2012 and the BLS or World Bank.[28] We first assign the closest price index to each commodity using BLS data based on the industry

[27] Also letter-case variations such as `Purchase obligations`, `purchase obligations`, etc.

[28] The Economic Census can be accessed on `https://www.census.gov/data/datasets/2012/econ/census/2012-manufacturing.html`.

code using price indexes by industry.[29] If there is no price, we manually assign the closest commodity based on the name on the Economic Census.[30]. Finally, for some commodities, only World Bank Commodity Data have a relevant price. [31]

The next set is to construct expenditure shares of each sector (NAICS-3) on all other sectors using the Economic Census Materials Consumed by Kind of Industry. For each 3-digit manufacturing sector, the input price index is the sum of the product of the price index of each commodity and its share in that sector.

[29] https://download.bls.gov/pub/time.series/pc/.

[30] See https://download.bls.gov/pub/time.series/wp/

[31] See https://www.worldbank.org/en/research/commodity-markets However, only 0.45% of the commodity prices we used were from the World Bank.

C Solution Algorithm - Second Stage

In this appendix, we explain the algorithm used to solve for the equilibrium in the second stage.

C.1 Flexible wage

We allow the economy to have flexible wages for $q \leq \bar{q}$. The steps below describe the algorithm used to solve the second stage conditional on $(q, r, K_s, \bar{m}_i^m, k_i^m \; \forall \, i)$.

1. Guess w, P^m, Y^m, Y^s
2. Solve for price index in services:
$$P^s = \frac{\sigma^s}{\sigma^s - 1} \left(\frac{q}{\gamma^s}\right)^{\gamma^s} \left(\frac{w}{1-\gamma^s}\right)^{1-\gamma^s} \frac{1}{Z^s (K^s)^{\frac{\varphi^s}{1-\varphi^s}}}$$
3. Economy-wide price index is
$$P = \left(\frac{P^m}{\alpha}\right)^{\alpha} \left(\frac{P^s}{1-\alpha}\right)^{1-\alpha}$$
4. Solve for firm profits in manufacturing and interest rates ($\forall \, i$):
$$\pi_i^m = \left(\frac{\sigma^m - 1}{\sigma^m}\right)^{\sigma^m} \frac{1}{\sigma^m - 1} (z_i)^{\sigma^m - 1} (k_i^m)^{\frac{(\sigma^m-1)\varphi^m}{1-\varphi^m}} \left(\frac{q}{\gamma^m}\right)^{\gamma^m} \left(\frac{w}{1-\gamma^m}\right)^{1-\gamma^m} (1+r_i^*)^{1-\sigma^m} \times$$
$$(P^m)^{\sigma^m} Y^m \left[\frac{1+\iota\sigma^m}{\sigma^m - 1}\right] + (\bar{m}_i^m [q - \bar{q}] - r k_i^m)(1 + r_i^*)$$
$$1 + r_i^* = (\pi_i^m / cogs_i^m)^{-\iota}$$
5. For firms in the service sector:
$$\Pi^s = \left(\frac{\sigma_s - 1}{\sigma_s}\right)^{\sigma^s} \frac{1}{\sigma_s - 1} (Z^s)^{\sigma^s - 1} (K^s)^{\frac{(\sigma_s - 1)\varphi^s}{1-\varphi^s}} \left(\frac{q}{\gamma^s}\right)^{\gamma^s} \left(\frac{w}{1-\gamma^s}\right)^{1-\gamma^s} (P^s)^{\sigma_s} Y^s$$
6. Compute aggregate profits as: $\Pi = \int \pi_i^m dH_m + \Pi^s$
7. Compute total factor productivities
$$(Z_k^m)^{\sigma^m - 1} = \int_i \left(\frac{z_i^m (k_i^m)^{\frac{\varphi^m}{1-\varphi^m}}}{(1+r_i^*)(1+\iota)}\right)^{\sigma^m - 1} dH^m \quad (Z_2^m)^{\sigma^m - 1} = \int_i \left(\frac{z_i^m (k_i^m)^{\frac{\varphi^m}{1-\varphi^m}}}{(1+r_i^*)(1+\iota)}\right)^{\sigma^m - 1} (1+r_i^*)(1+\iota) dH^m$$
8. For wages, use the price index definition for the manufacturing sector
$$w = \left(\frac{\sigma^m - 1}{\sigma^m}\right)^{\frac{1}{1-\gamma^m}} \left(\frac{q}{\gamma^m}\right)^{-\frac{\gamma^m}{1-\gamma^m}} (Z_k^m)^{\frac{1}{1-\gamma^m}} (1-\gamma^m) (P^m)^{\frac{1}{1-\gamma^m}}$$
9. Labor supply: $L^s = \left(\frac{w}{P\xi}\right)^{\frac{1}{\eta}}$
10. For output Y^m, use $L^d = L^s$, where:
$$L^d = \left(\frac{\sigma^m - 1}{\sigma^m}\right)^{\sigma^m} \left(\frac{q}{\gamma^m}\right)^{\gamma^m(1-\sigma^m)} \left(\frac{w}{1-\gamma^m}\right)^{(1-\gamma^m)(1-\sigma^m)} (P^m)^{\sigma^m} Y^m \frac{1-\gamma^m}{w} (Z_2^m)^{\sigma^m - 1} +$$
$$\left(\frac{\sigma^s - 1}{\sigma^s}\right)^{\sigma^s} \left(\frac{q}{\gamma^s}\right)^{\gamma^s(1-\sigma^s)} \left(\frac{w}{1-\gamma^s}\right)^{(1-\gamma^s)(1-\sigma^s)} (P^s)^{\sigma^s} Y^s \frac{1-\gamma^s}{w} (Z^s (K^s)^{\frac{\varphi^s}{1-\varphi^s}})^{\sigma^s - 1}$$
11. External Financing reimbursed to the consumer
$$EF = P_m Y_m \frac{\sigma_m - 1}{\sigma_m} \frac{1}{(Z_m^k)^{\sigma_m - 1}} \int_i \left(\frac{z_i^m (k_i^m)^{\frac{\varphi^m}{1-\varphi^m}}}{(1+r_i^*)(1+\iota)}\right)^{\sigma_m - 1} \frac{r_i^*}{(1+r_i^*)(1+\iota)} - \int \bar{m}_i^m [q - \bar{q}] r_i^* dH_m + \int_i r k_i^m r_i^* dH_m$$

12. Solving for consumption:

$$C = \frac{wL^d + \Pi - \kappa^A + EF + rK_s + r\sum_i k_i^m dH_m}{P}$$

$$C^m = \alpha \frac{PC}{P^m}$$

$$C^s = (1-\alpha)\frac{PC}{P^s}$$

where $\kappa^A \equiv \int_i \left(\kappa_a \mathbb{I}_{\bar{m}_i^m > 0} + \kappa_b \bar{m}_i^m\right) dH^m$

13. Total output: $Y = P^m Y^m + P^s Y^s$

14. For P^m, use market clearing for the final good tradeable good:

$$Y = C + P^{-\nu}(P^*)^{\nu}\zeta Y^*$$

15. For Y^s use equilibirum in non-tradeables $Y^s = C^s$

16. Iterate over equations for w, Y^m, P^m and Y^s

C.2 Fixed wage

For $q > \bar{q}$, we include downward nominal wage rigidities that imply $w = \bar{w}$, where $\bar{w} = w(\mathbb{E}_q[q])$. The steps below describe the algorithm used to solve the second stage conditional on $(q, r, K_s, \bar{m}_i^m, k_i^m \;\forall\, i)$.

1. Guess u, P^m, Y^m, Y^s

2. Solve for the price index in services:

$$P^s = \frac{\sigma^s}{\sigma^s - 1}\left(\frac{q}{\gamma^s}\right)^{\gamma^s}\left(\frac{\bar{w}}{1-\gamma^s}\right)^{1-\gamma^s}\frac{1}{Z^s (K^s)^{\frac{\varphi^s}{1-\varphi^s}}}$$

3. Economy-wide price index is

$$P = \left(\frac{P^m}{\alpha}\right)^{\alpha}\left(\frac{P^s}{1-\alpha}\right)^{1-\alpha}$$

4. Solve for firm profits in manufacturing and interest rates ($\forall\, i$):

$$\pi_i^m = \left(\frac{\sigma^m - 1}{\sigma^m}\right)^{\sigma^m}\frac{1}{\sigma^m - 1}(z_i)^{\sigma^m - 1}(k_i^m)^{\frac{(\sigma^m-1)\varphi^m}{1-\varphi^m}}\left(\frac{q}{\gamma^m}\right)^{\gamma^m}\left(\frac{w}{1-\gamma^m}\right)^{1-\gamma^m}(1+r_i^*)^{1-\sigma^m} \times$$
$$(P^m)^{\sigma^m} Y^m \left[\frac{1+\iota\sigma^m}{\sigma^m - 1}\right] + (\bar{m}_i^m[q-\bar{q}] - rk_i^m)(1+r_i^*)$$
$$1 + r_i^* = (\pi_i^m / cogs_i^m)^{-\iota}$$

5. For firms in the service sector:

$$\Pi^s = \left(\frac{\sigma^s - 1}{\sigma^s}\right)^{\sigma^s}\frac{1}{\sigma^s - 1}(Z^s (K^s)^{\frac{\varphi_s}{1-\varphi_s}})^{\sigma^s - 1}\left(\frac{q}{\gamma^s}\right)^{\gamma^s}\left(\frac{\bar{w}}{1-\gamma^s}\right)^{1-\gamma^s}(P^s)^{\sigma^s} Y^s$$

6. Compute aggregate profits as: $\Pi = \int \pi_i^m dH_m + \pi^s$

7. Compute total factor productivities

$$(Z_k^m)^{\sigma^m - 1} = \int_i \left(\frac{z_i^m (k_i^m)^{\frac{\varphi^m}{1-\varphi^m}}}{(1+r_i^*)(1+\iota)}\right)^{\sigma^m - 1} dH^m \quad (Z_2^m)^{\sigma^m - 1} = \int_i \left(\frac{z_i^m (k_i^m)^{\frac{\varphi^m}{1-\varphi^m}}}{(1+r_i^*)(1+\iota)}\right)^{\sigma^m - 1}(1+r_i^*)(1+\iota) dH^m$$

8. Labor supply: $L^s = \left(\frac{\bar{w}}{P\xi}\right)^{\frac{1}{\eta}}$

9. For output Y^m, use $L^s = L^d + u$, where:

$$L^d = \left(\frac{\sigma^m - 1}{\sigma^m}\right)^{\sigma^m}\left(\frac{q}{\gamma^m}\right)^{\gamma^m(1-\sigma^m)}\left(\frac{\bar{w}}{1-\gamma^m}\right)^{(1-\gamma^m)(1-\sigma^m)}(P^m)^{\sigma} Y^m \frac{1-\gamma^m}{\bar{w}}(Z_2^m)^{\sigma^m - 1} +$$
$$\left(\frac{\sigma^s - 1}{\sigma^s}\right)^{\sigma^s}\left(\frac{q}{\gamma^s}\right)^{\gamma^s(1-\sigma^s)}\left(\frac{\bar{w}}{1-\gamma^s}\right)^{(1-\gamma^s)(1-\sigma^s)}(P^s)^{\sigma^s} Y^s \frac{1-\gamma^s}{\bar{w}}(Z^s (K^s)^{\frac{\varphi^s}{1-\varphi^s}})^{\sigma^s - 1}$$

10. For P^m, use:

$$P^m = \frac{\sigma^m}{\sigma^m - 1} \left(\frac{q}{\gamma^m}\right)^{\gamma^m} \left(\frac{\bar{w}}{1-\gamma^m}\right)^{1-\gamma^m} \frac{1}{Z_k^m}$$

11. External Financing reimbursed to the consumer

$$EF = P_m Y_m \frac{\sigma_m - 1}{\sigma_m} \frac{1}{(Z_m^k)^{\sigma_m - 1}} \int_i \left(\frac{z_i^m (k_i^m)^{\frac{\varphi^m}{1-\varphi^m}}}{(1+r_i^*)(1+\iota)} \right)^{\sigma_m - 1} \frac{r_i^*}{(1+r_i^*)(1+\iota)} - \int \bar{m}_i^m [q - \bar{q}] r_i^* dH_m + \int_i r k_i^m r_i^* dH_m$$

12. Solving for consumption:

$$C = \frac{wL^d + \Pi - \kappa^A + EF + rK^s + r\int_i k_i^m dH_m}{P}$$

$$C^m = \alpha \frac{PC}{P^m}$$

$$C^s = (1-\alpha)\frac{PC}{P^s}$$

where $\kappa^A \equiv \int_i \left(\kappa_a \mathbb{I}_{\bar{m}_i^m > 0} + \kappa_b \bar{m}_i^m \right) dH^m$

13. Total output: $Y = P^m Y^m + P^s Y^s$

14. For unemployment u, use:

$$Y = C + P^{-\nu} (P^*)^{\nu} \zeta Y^*$$

15. For Y^s, use equilibirum in non-tradeables $Y^s = C^s$

16. Iterate over equations for Y^m, P^m, u and Y^s

D Solution Algorithm–First Stage

In this appendix, we explain the algorithm used to solve for the equilibrium in the first stage. In summary, we choose purchase obligation quantities using an iterated method.

1. Solve equilibrium in second stage assuming $\bar{m}_i^m = 0$ for all firms. This pins down aggregate variables.
2. Solve for $\bar{m}_i^m$ using equations (24) and (**??**) conditional on aggregate variables of previous point.
3. Update second stage using new PO quantities $\bar{m}_i^m$. In particular, find aggregate variables for new vector of PO quantities.
4. Repeat stages 2 and 4 until the PO quantity vector converges.

E Proofs

Proposition 1

Proof. In the 2nd Stage, firm i (in manufacturing) has some capital k_i^m and supply contracts $\bar{m}_i^m$. Firms cost minimization problem is

$$\min_{l_i^m, m_i^m} wl_i^m + qm_i^m + \bar{q}\bar{m}_i^m + rk_i^m \quad \text{s.t.} y_i^m \leq z_i (l_i^m)^{(1-\gamma^m)} (m_i^m + \bar{m}_i^m)^{\gamma^m} (k_i^m)^{\frac{\varphi^m}{1-\varphi^m}} x`$$

FOCS imply:

$$\begin{aligned} (l_i^m): \quad & z_i (l_i^m)^{(1-\gamma^m)-1} (m_i^m + \bar{m}_i^m)^{\gamma^m} (k_i^m)^{\frac{\varphi^m}{1-\varphi^m}} (1-\gamma^m) = w \\ (m_i^m): \quad & z_i (l_i^m)^{(1-\gamma^m)} (m_i^m + \bar{m}_i^m)^{\gamma^m - 1} (k_i^m)^{\frac{\varphi^m}{1-\varphi^m}} \gamma^m = q \end{aligned}$$

Ratio implies:

$$\frac{w}{q} = \frac{1-\gamma^m}{\gamma^m} \frac{m_i^m + \bar{m}_i^m}{l_i^m} \rightarrow m_i^m + \bar{m}_i^m = l_i^m \frac{w}{1-\gamma^m} \frac{\gamma^m}{q}$$

Replacing in constraint:

$$\begin{aligned} \frac{y_i^m}{z_i} &= (l_i^m)^{(1-\gamma^m)} \left(\frac{w}{1-\gamma^m}\right)^{\gamma^m} \left(\frac{\gamma^m}{q}\right)^{\gamma^m} (l_i^m)^{\gamma^m} (k_i^m)^{\frac{\varphi^m}{1-\varphi^m}} \\ l_i^m &= \frac{y_i^m}{z_i} \left(\frac{1-\gamma^m}{w}\right)^{\gamma^m} \left(\frac{q}{\gamma^m}\right)^{\gamma^m} (l_i^m)^{\gamma^m} (k_i^m)^{-\frac{\varphi^m}{1-\varphi^m}} \end{aligned}$$

Analogous for materials:

$$m_i^m = \frac{y_i^m}{z_i} \left(\frac{w}{1-\gamma^m}\right)^{1-\gamma^m} \left(\frac{q}{\gamma^m}\right)^{\gamma^m} (k_i^m)^{\frac{-\varphi^m}{1-\varphi^m}} \frac{\gamma^m}{q} - \bar{m}_i^m$$

Costs:

$$cogs_i^m = \frac{y_i^m}{z_i} \left(\frac{w}{1-\gamma^m}\right)^{1-\gamma^m} \left(\frac{q}{\gamma^m}\right)^{\gamma^m} (k_i^m)^{\frac{-\varphi^m}{1-\varphi^m}} + \bar{m}_i^m [\bar{q} - q] + rk_i^m$$

□

Proposition 2

Proof. Profit Maximization for firm i in manufacturing is

$$
\begin{aligned}
&\max_{p_i^m} p_i^m y_i^m - cogs_i^m(1+r_i^*) \quad \text{s.t.:}\\
&y_i^m = \left(\frac{p_i^m}{P_m}\right)^{\sigma^m} Y_m\\
&1+r_i^* = \left(\frac{\pi_m^m}{cogs_i^m}\right)^{-\iota}
\end{aligned}
$$

FOC:

$$
(1-\sigma^m)(p_i^m)^{-\sigma^m} P_m^{\sigma^m} Y_m + -\sigma^m (p_i^m)^{-\sigma^m-1} P_m^{\sigma^m} Y_m \frac{MC_m}{z_i (k_i^m)^{\frac{\varphi^m}{1-\varphi^m}}}(1+r_i^*) - cogs_i^m \frac{\partial(1+r_i^*)}{\partial p_i^m} = 0
$$

Solving for financial constraint derivative:

$$
\begin{aligned}
\frac{\partial(1+r_i^*)}{\partial p_i^m} &= (-\iota)\left(\frac{\pi_i^m}{cogs_i^m}/\bar{B}\right)^{-\iota-1} \bar{B}\, \frac{\overbrace{\frac{\partial \pi_i}{\partial p_i^m}}^{0} cogs_i^m - \frac{\partial cogs_i}{\partial p_i^m}\pi_i^m}{cogs_i^2}\\
&= \iota(1+r_i^*)\frac{1}{cogs_i}(-\sigma^m)(p_i^m)^{-\sigma^m-1} P_m^{\sigma^m} Y_m \frac{MC_m}{z_i (k_i^m)^{\frac{\varphi^m}{1-\varphi^m}}}
\end{aligned}
$$

Replacing in FOC and solving for price rule:

$$
\begin{aligned}
&(1-\sigma^m)(p_i^m)^{-\sigma^m} P_m^{\sigma^m} Y_m - \sigma^m (p_i^m)^{-\sigma^m-1} P_m^{\sigma^m} Y_m \frac{MC_m}{z_i (k_i^m)^{\frac{\varphi^m}{1-\varphi^m}}}(1+r_i^*)(1+\iota) = 0\\
&\sigma^m (p_i^m)^{-1} \frac{MC_m}{z_i (k_i^m)^{\frac{\varphi^m}{1-\varphi^m}}}(1+r_i^*)(1+\iota) = (1-\sigma^m)\\
&p_i^m = \frac{\sigma^m}{\sigma^m-1}\frac{MC_m}{z_i (k_i^m)^{\frac{\varphi^m}{1-\varphi^m}}}(1+r_i^*)(1+\iota)
\end{aligned}
$$

□

Proposition 3

Proof. Need to solve for ratio $\pi_m^i/cogs_i^m$. Notice for $cogs_i^m$:

$$
\begin{aligned}
cogs_i^m &= \frac{y_i^m}{z_i (k_i^m)^{\frac{-\varphi^m}{1-\varphi^m}}}\left(\frac{w}{1-\gamma^m}\right)^{1-\gamma^m}\left(\frac{q}{\gamma^m}\right)^{\gamma^m} - \bar{m}_i^m[q-\bar{q}] + r k_i^m\\
&= \left(\frac{\sigma^m}{\sigma^m-1}\right)^{-\sigma^m}\left(\frac{MC_m}{z_i (k_i^m)^{\frac{\varphi^m}{1-\varphi^m}}}\right)^{1-\sigma^m} P_m^{\sigma^m} Y_m (1+r_i^*)^{-\sigma^m}(1+\iota)^{-\sigma^m} - \bar{m}_i^m[q-\bar{q}] + r k_i^m
\end{aligned}
$$

For profits:

$$
\begin{aligned}
\pi_i^m &= p_i^m y_i^m - cogs_i^m(1+r_i^*)\\
&= \frac{\sigma^m}{\sigma^m-1}\left(\frac{\sigma^m}{\sigma^m-1}\right)^{-\sigma^m} P_m^{\sigma^m} Y_m \left(\frac{MC_m}{z_i (k_i^m)^{\frac{\varphi^m}{1-\varphi^m}}}\right)^{1-\sigma^m}(1+r_i^*)^{1-\sigma^m}(1+\iota)^{1-\sigma^m} - cogs_i^m(1+r_i^*)\\
&= \left(\frac{\sigma^m}{\sigma^m-1}\right)^{-\sigma^m} P_m^{\sigma^m} Y_m \left(\frac{MC_m}{z_i (k_i^m)^{\frac{\varphi^m}{1-\varphi^m}}}\right)^{1-\sigma^m}(1+r_i^*)^{1-\sigma^m}(1+\iota)^{-\sigma^m}\left[\frac{\sigma^m}{\sigma^m-1}(1+\iota)-1\right]\\
&= \left(\frac{\sigma^m-1}{\sigma^m}\right)^{\sigma^m}\left(\frac{MC_m}{z_i (k_i^m)^{\frac{\varphi^m}{1-\varphi^m}}}\right)^{1-\sigma^m} P_m^{\sigma^m} Y_m (1+r_i^*)^{1-\sigma^m}(1+\iota)^{-\sigma^m}\left[\frac{1+\iota\sigma^m}{\sigma^m-1}\right]\\
&\quad + (\bar{m}_i^m[q-\bar{q}] - r k_i^m)(1+r_i^*)
\end{aligned}
$$

□

Definition(Discount Factor) - Details

A standard problem where households choose consumption and shares in the aggregate mutual fund is:

$$\max_{C(q_t),\theta(q_{t+1})} \mathbb{E}_0 \sum_t \beta^t U(C(q_t)) \quad s.t. \quad P(q_t)C(q_t) + \theta(q_{t+1})V(q_t) = I(q_t) + \theta(q_t)[V(q_t) + D(q_t)]$$

where q is commodity spot price, V is company market value, D dividends, I household nominal income, C consumption and P consumption units price. First order conditions imply:

$$\begin{aligned}(C(q_t)): &\quad \beta^t U'(C(q_t) = \mu(q_t)P(q_t)\\ (\theta_{t+1}): &\quad \mu_t V_t = \mathbb{E}_t\big[\mu_{t+1}\left[V_{t+1} + D_{t+1}\right]\big]\end{aligned}$$

where μ is the Lagrange multiplier of the problem. Rearranging yields:

$$V(q_t) = \mathbb{E}_t\left[\frac{\beta U'(C(q_{t+1}))/P(q_{t+1})}{U'q(C_t))/P(q_t)}\left[V(q_{t+1}) + D(q_{t+1})\right]\right]$$

Define the discount factor $\Lambda(q_{t+1}) = \frac{\beta U'(C(q_{t+1}))/P(q_{t+1})}{U'(C(q_t))/P(q_t)}$. To finish the proof we set variables at t corresponding to mean spot prices $\bar{q}$ because we analyze commodity price shocks starting at the mean, we drop $t+1$ subscripts and assume $\beta = 1$ because we have only one period in the model. Therefore $\Lambda(q) = \frac{U'(C(q))/P(q)}{U'(C(\bar{q}))/P(\bar{q})}$.

Proposition 5

Proof.

$$\begin{aligned}\frac{\partial \pi_i^m}{\partial \bar{m}_i^m} &= [q-\bar{q}](1+r_i^*) + \\ &\left[\left(\frac{\sigma^m - 1}{\sigma^m}\right)^{\sigma^m}(MC_m)^{1-\sigma^m}(z_i)^{\sigma^m-1}(k_i^m)^{\frac{(\sigma^m-1)\varphi^m}{1-\varphi^m}} \times \right.\\ &\left. P_m^{\sigma^m} Y_m \frac{1-\sigma^m}{1+r_i^*}(1+r_i^*)^{1-\sigma^m}(1+\iota)^{-\sigma^m}\left[\frac{1+\iota\sigma^m}{\sigma^m-1}\right] - rk_i^m + \bar{m}_i^m[q-\bar{q}]\right]\frac{\partial 1+r_i^*}{\partial k_i^m}\\ \frac{\partial \pi_i^m}{\partial \bar{m}_i^m} &\equiv \Omega_{11} + \Omega_{12}\frac{\partial 1+r_i^*}{\partial \bar{m}_i^m}\end{aligned}$$

Constraints derivative:

$$\begin{aligned}1+r_i^* &= \left(\frac{\pi_i^m}{cogs_i^m}\right)^{-\iota}\\ \frac{\partial(1+r_i^*)}{\partial \bar{m}_i^m} &= (-\iota)\left(\frac{\pi_i^m}{cogs_i^m}\right)^{-\iota-1}\frac{\frac{\partial \pi_i}{\partial \bar{m}_i^m}cogs_i^m - \frac{\partial cogs_i}{\partial \bar{m}_i^m}\pi_i^m}{cogs_i^2}\\ &= -\iota(1+r_i^*)\left[\frac{\partial \pi_i^m}{\partial \bar{m}_i^m}\frac{1}{\pi_i^m} - \frac{\partial cogs_i^m}{\partial \bar{m}_i^m}\frac{1}{cogs_i^m}\right]\end{aligned}$$

Cogs derivative:

$$\begin{aligned}cogs_i^m &= \left(\frac{\sigma^m-1}{\sigma^m}\right)^{\sigma^m}(MC_m)^{1-\sigma^m}(z_i)^{\sigma^m-1}(k_i^m)^{\frac{(\sigma^m-1)\varphi^m}{1-\varphi^m}}(1+r_i^*)^{-\sigma^m}\\ &\quad (1+\iota)^{-\sigma^m}P_m^{\sigma^m}Y_m + rk_i^m - \bar{m}_i^m[q-\bar{q}]\\ \frac{\partial cogs_i^m}{\partial \bar{m}_i^m} &= -[q-\bar{q}] + \\ &\left[\left(\frac{\sigma^m-1}{\sigma^m}\right)^{\sigma^m}(MC_m)^{1-\sigma^m}(z_i)^{\sigma^m-1}(k_i^m)^{\frac{(\sigma^m-1)\varphi^m}{1-\varphi^m}} \times \right.\\ &\left. P_m^{\sigma^m}Y_m\frac{-\sigma^m}{1+r_i^*}(1+r_i^*)^{-\sigma^m}(1+\iota)^{-\sigma^m}\right]\frac{\partial 1+r_i^*}{\partial \bar{m}_i^m}\\ \frac{\partial cogs_i^m}{\partial \bar{m}_i^m} &\equiv \Omega_{21} + \Omega_{22}\frac{\partial 1+r_i^*}{\partial \bar{m}_i^m}\end{aligned}$$

Therefore, we have a system of equations:

$$\frac{\partial(1+r_i^*)}{\partial\bar{m}_i^m} = -\iota(1+r_i^*)\left[\frac{\partial\pi_i^m}{\partial\bar{m}_i^m}\frac{1}{\pi_i^m} - \frac{\partial cogs_i^m}{\partial\bar{m}_i^m}\frac{1}{cogs_i^m}\right]$$
$$\frac{\partial\pi_i^m}{\partial\bar{m}_i^m} = \Omega_{11} + \Omega_{12}\frac{\partial(1+r_i^*)}{\partial\bar{m}_i^m}$$
$$\frac{\partial cogs_i^m}{\partial\bar{m}_i^m} = \Omega_{21} + \Omega_{22}\frac{\partial(1+r_i^*)}{\partial\bar{m}_i^m}$$

Solving:

$$\frac{\partial(1+r_i^*)}{\partial\bar{m}_i^m} = -\iota(1+r_i^*)\left[\frac{\Theta_{11}}{\pi_i^m} + \frac{\Theta_{12}}{\pi_i^m}\frac{\partial(1+r_i^*)}{\partial\bar{m}_i^m} - \frac{\Theta_{21}}{cogs_i^m} - \frac{\Theta_{22}}{cogs_i^m}\frac{\partial(1+r_i^*)}{\partial\bar{m}_i^m}\right]$$
$$\frac{\partial(1+r_i^*)}{\partial\bar{m}_i^m} = -\iota(1+r_i^*)\frac{\frac{\Omega_{11}}{\pi_i^m} - \frac{\Omega_{21}}{cogs_i^m}}{1+\iota(1+r_i^*)\left[\frac{\Omega_{12}}{\pi_i^m} - \frac{\Omega_{22}}{cogs}\right]}$$

Rewriting:

$$\Omega_{11} = (q-\bar{q})(1+r_i^*)$$

$$\Omega_{12} = (\pi_i^m + rk_i^m(1+r_i^*) - \bar{m}[q-\bar{q}](1+r_i^*))\frac{1-\sigma^m}{1+r_i^*} - rk_i^m + \bar{m}[q-\bar{q}]$$
$$\Omega_{12} = \frac{\pi_i^m}{1+r_i^*}(1-\sigma^m) - rk_i^m\sigma^m + \bar{m}[q-\bar{q}]\sigma^m$$

$$\Omega_{21} = -(q-\bar{q})$$
$$\Omega_{22} = (cogs_i^m - rk_i^m + \bar{m}[q-\bar{q}])\frac{-\sigma^m}{1+r_i^*}$$
$$\Omega_{22} = \frac{cogs_i^m}{1+r_i^*}(-\sigma^m) + rk_i^m\frac{\sigma^m}{1+r_i^*} - \bar{m}[q-\bar{q}]\frac{\sigma^m}{1+r_i^*}$$

Then constraint derivative:

$$\frac{\partial(1+r_i^*)}{\partial\bar{m}_i^m} = -\iota(1+r_i^*)\frac{\frac{\Omega_{11}}{\pi_i^m} - \frac{\Omega_{21}}{cogs_i^m}}{1+\iota(1+r_i^*)\left[\frac{\Omega_{12}}{\pi_i^m} - \frac{\Omega_{22}}{cogs}\right]}$$

$$\text{Numerator: } -\iota(1+r_i^*)\left[(q-\bar{q})\frac{1+r_i^*}{\pi_i^m} + \frac{q-\bar{q}}{cogs_i^m}\right]$$
$$= -\iota(1+r_i^*)^2\frac{q-\bar{q}}{\pi_i^m}\left[1+\frac{\pi_i^m}{cogs_i^m(1+r_i^*)}\right]$$

$$\text{Denominator: } 1+\iota(1+r_i^*)[\frac{1-\sigma^m}{1+r_i^*} - \frac{rk_i^m}{\pi_i^m} + \bar{m}\frac{q-\bar{q}}{\pi_i^m}\sigma^m + \frac{\sigma^m}{1+r_i^m} - \frac{rk_i^m}{(1+r_i^*)(cogs_i^m)}$$
$$+\bar{m}\frac{[q-\bar{q}]}{(1+r_i^*)cogs_i^m}\sigma^m]$$
$$= 1+\iota+\iota\sigma^m\frac{1+r_i^*}{\pi_m^i}\left[1+\frac{\pi_i^m}{cogs_i^m(1+r_i^*)}\right][\bar{m}_i^m(q-\bar{q}) - rk_i^m]$$

Hence:

$$\frac{\partial(1+r_i^*)}{\partial\bar{m}_i^m} = \frac{-\iota(1+r_i^*)^2\frac{q-\bar{q}}{\pi_i^m}\left[1+\frac{\pi_i^m}{cogs_i^m(1+r_i^*)}\right]}{1+\iota+\iota\sigma^m\frac{1+r_i^*}{\pi_m^i}\left[1+\frac{\pi_i^m}{cogs_i^m(1+r_i^*)}\right][\bar{m}_i^m(q-\bar{q}) - rk_i^m]}$$

Back in FOC:

$$
\begin{aligned}
\frac{\partial \pi_i^m}{\partial \bar{m}_i^m} &\equiv \Omega_{11} + \Omega_{12}\frac{\partial 1+r_i^*}{\partial \bar{m}_i^m} \\
&= \frac{(q-\bar{q})(1+r_i^*)\left\{1+\iota+\iota\sigma^m \frac{1+r_i^*}{\pi_m^i}\left[1+\frac{\pi_i^m}{cogs_i^m(1+r_i^*)}\right]\left[\bar{m}_i^m(q-\bar{q})-rk_i^m\right]\right\}}{\text{Denominator}} + \\
&\quad \left\{\frac{\pi_i^m}{1+r_i^*}(1-\sigma^m) - rk_i^m\sigma^m + \bar{m}[q-\bar{q}]\sigma^m\right\}\frac{-\iota(1+r_i^*)^2\frac{q-\bar{q}}{\pi_i^m}\left[1+\frac{\pi_i^m}{cogs_i^m(1+r_i^*)}\right]}{\text{Denominator}} \\
&= \frac{\left\{(q-\bar{q})(1+r_i^*)(1+\iota)+(q-\bar{q})\iota\sigma^m\frac{(1+r_i^*)^2}{\pi_m^i}\left[1+\frac{\pi_i^m}{cogs_i^m(1+r_i^*)}\right]\left[\bar{m}_i^m(q-\bar{q})-rk_i^m\right]\right\}}{\text{Denominator}} + \\
&\quad \left\{\frac{\pi_i^m}{1+r_i^*}\frac{(1-\sigma^m)}{\sigma^m} - rk_i^m + \bar{m}[q-\bar{q}]\right\}\frac{-\iota\sigma^m(1+r_i^*)^2\frac{q-\bar{q}}{\pi_i^m}\left[1+\frac{\pi_i^m}{cogs_i^m(1+r_i^*)}\right]}{\text{Denominator}} \\
&= \frac{\left\{(q-\bar{q})(1+r_i^*)(1+\iota)\right\}}{\text{Denominator}} + \left\{\frac{\pi_i^m}{1+r_i^*}\frac{(1-\sigma^m)}{\sigma^m}\right\}\frac{-\iota\sigma^m(1+r_i^*)^2\frac{q-\bar{q}}{\pi_i^m}\left[1+\frac{\pi_i^m}{cogs_i^m(1+r_i^*)}\right]}{\text{Denominator}}
\end{aligned}
$$

Finally:

$$
\frac{\partial \pi_i^m}{\partial \bar{m}_i^m} = (q-\bar{q})(1+r_i^*)\frac{1+\iota+\iota(\sigma^m-1)\left[1+\frac{\pi_i^m}{cogs_i^m(1+r_i^*)}\right]}{1+\iota+\iota\sigma^m\frac{1+r_i^*}{\pi_m^i}\left[1+\frac{\pi_i^m}{cogs_i^m(1+r_i^*)}\right]\left[\bar{m}_i^m(q-\bar{q})-rk_i^m\right]}
$$

$\square$

Proposition 6

Proof. Notice $Z_k^{\sigma^m-1} = \int_i \left(\frac{z_i(k_i^m)^{\frac{\varphi^m}{\varphi^m-1}}}{(1+\iota)(1+r_i^*)}\right)^{\sigma^m-1} dH(z)$. Applying hat algebra after a shock in spot prices (i.e. $\hat{X} \equiv \frac{X'}{X}$):

$$
\begin{aligned}
(\hat{Z}_k)^{\sigma^m-1} &= \frac{\int_i \left(\frac{z_i(k_i^m)^{\frac{\varphi^m}{\varphi^m-1}}}{(1+\iota)(1+r_i^*)'}\right)^{\sigma^m-1} dH(z)}{\int_i \left(\frac{z_i(k_i^m)^{\frac{\varphi^m}{\varphi^m-1}}}{(1+\iota)(1+r_i^*)}\right)^{\sigma^m-1} dH(z)} \\
&= \int_i \frac{\left(\frac{z_i(k_i^m)^{\frac{\varphi^m}{\varphi^m-1}}}{(1+\iota)(1+r_i^*)(\widehat{1+r_i^*})}\right)^{\sigma^m-1} dH(z)}{Z_k^{\sigma^m-1}}
\end{aligned}
$$

Finally, we use $\widehat{1+r_i^*} \approx \mathrm{d}\log(1+r_i^*)$, so

$$
\mathrm{d}\log Z_k \approx -\int_i \frac{\left(\frac{z_i(k_i^m)^{\frac{\varphi^m}{\varphi^m-1}}}{(1+\iota)(1+r_i^*)}\right)^{\sigma^m-1} \mathrm{d}\log(1+r_i^*)dH(z)}{Z_k^{\sigma^m-1}}
$$

$\square$